RMO 2025 Round 1

1. Find the value of

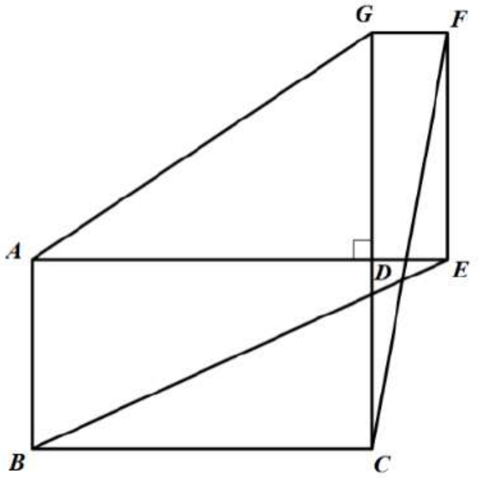
$$(1+\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\cdots+\frac{2023}{2025})(\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\cdots+\frac{2021}{2023})-(\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\cdots+\frac{2023}{2025})(1+\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\cdots+\frac{2021}{2023})$$

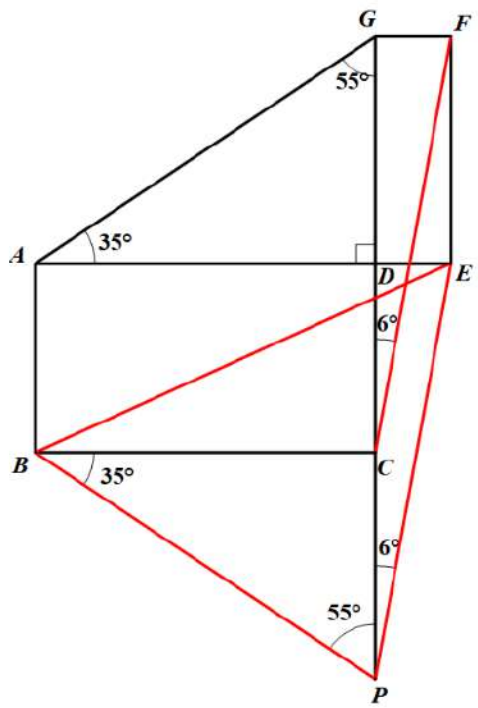
【Answer】 $-\frac{2023}{2025}$

【Solution】 Let $\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\cdots+\frac{2021}{2023}$ be x.

$$\begin{aligned}\text{Ans} &= (1+x+\frac{2023}{2025})\cdot x-(x+\frac{2023}{2025})(1+x)\\ &= x+x^2+\frac{2023}{2025}x-x-\frac{2023}{2025}-x^2-\frac{2023}{2025}x\\ &= -\frac{2023}{2025}\end{aligned}$$

2. Given that both $ABCD$ and $DEFG$ are rectangles, we know that $\angle FCG=6°$, $\angle DAG=35°$, and $BE=CF$. Find the angle of $\angle AEB$.





【Answer】 26°

【Solution】 Translate CF downwards until F coincides with E.

Now $PE=CF=BE$,

So $\triangle BPE$ is an isosceles triangle, $\angle EBP=\angle EPB$

$GD=PC$, $\triangle ADG$ is same with $\triangle BCP$.

So $\angle CBP=\angle DAG=35°$

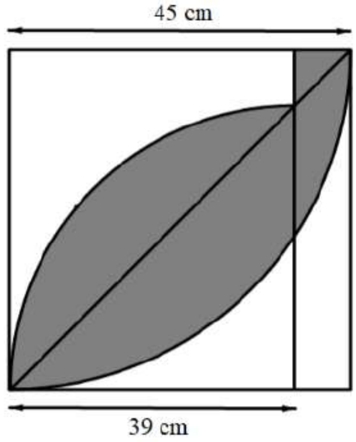
$\angle BPC=\angle AGD=90°-35°=55°$

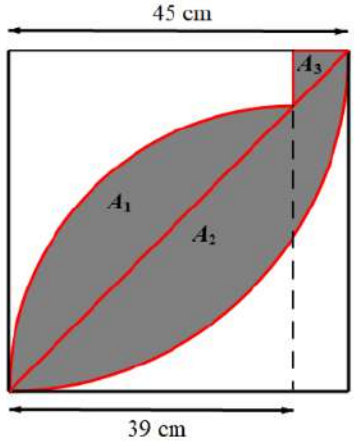
$\angle EPC=\angle FCG=6°$

So $\angle EBP=\angle EPB=55°+6°=61°$

Then $\angle AEB=\angle CBE=61°-35°=26°$

3. Given that the total area of the large square is 2025, and the line at the base is 39cm as shown in the figure, find the area of the shaded region.





【Answer】 904.5(if $\pi = 3$)

【Solution】 $A_1 = \frac{1}{4}\pi \cdot 39^2 - \frac{1}{2} \cdot 39^2 = 380.25$

$A_2 = \frac{1}{4}\pi \cdot 45^2 - \frac{1}{2} \cdot 45^2 = 506.25$

$A_3 = \frac{(45-39)^2}{2} = 18$

So the total area of shaded parts $= 380.25 + 506.25 + 18 = 904.5$

4. We have a number that follows the following pattern, 122333444455555… Find the 2025th digit.

【Answer】 5

【Solution】 1-9: each number has only one digit.

10-99: each number has two digit.

Suppose the last number is n

Total digits: $(1+2+3+\cdots+n)\times 2-(1+2+3+\cdots+9) \rightarrow 2025$.

$n \times (n+1) \times 2 \div 2 - 45 \rightarrow 2025$

$n \times (n+1) \rightarrow 2070$.

$45 \times 46 = 2070$

so the last number is 45, the last digit is 5.

5. How many 3-digit integers are there, such that its tens place is the average of the other places.

【Answer】 45

【Solution】 Let the three-digit number be represented as ABC. 2B = A+C

(1) B = 0, so A= 0, C = 0 (×)

(2) B = 1, so A + C = 2, A = 2, C = 0
A=1, C=1

(3) B = 2, so A+C=4, A=4, C=0
A=3, C=1
A=2, C=2
A=1, C=3
……

Total $2+4+6+8+9+7+5+3+1=45$.

6. A and B are walking from place X to place Y. A takes 2 hours and 24 minutes to walk from X to Y and B takes 3 hours to walk from X to Y. Both A and B started walking at 8am. If A turns back immediately when reaching place Y, what time did A and B meet?

【Answer】 10:40

【Solution】 For both A and B, they covered the same distance between X and Y.

The ratio of their time taken is $2h24\min:3h=\frac{12}{5}h:3h=4:5$. Therefore, the ratio of their speeds is 5 : 4.

As shown in the figure above, A and B both depart from point X and meet at a certain point. The time they each take to reach the meeting point is the same. Since their speed ratio is 5:4, the distance ratio is also 5:4.

So, the total distance of two XYs is: 5u + 4u = 9u
The distance of one XY is: 9u ÷ 2 = 4.5u

Therefore, the time required in the question is: $3\div4.5\times4=2\frac{2}{3}h=2h40\min$. 8:00-10:40

7. $\frac{1008}{24}+\frac{1008}{40}+\frac{1008}{60}+\frac{1008}{84}+...+\frac{1008}{264}=?$

【Answer】 126

【Solution】

$$\frac{1008}{24}+\frac{1008}{40}+\frac{1008}{60}+\frac{1008}{84}+...+\frac{1008}{264}$$
$$=\frac{504}{12}+\frac{504}{20}+\frac{504}{30}+\frac{504}{42}+\cdots+\frac{504}{132}$$
$$=504(\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\cdots+\frac{1}{132})$$
$$=504(\frac{1}{3\times4}+\frac{1}{4\times5}+\frac{1}{5\times6}+\cdots+\frac{1}{11\times12})$$
$$=504(\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+\cdots+\frac{1}{11}-\frac{1}{12})$$
$$=504(\frac{1}{3}-\frac{1}{12})$$
$$=504\times\frac{1}{4}$$
$$=126$$

8. Alex has 42 candies. He gave a prime number of x candies to another person, and he had a prime number of candies left. He then gave a prime number of y candies to another person, and he had a prime number of candies left. Lastly, he gave a prime number of z candies to another person, and he had a non-zero even number of candies left. How many possibilities are there for the candies that he gave out?
(x, y, z) are not necessarily distinct.

【Answer】 16

【Solution】 **1st time**: split 42 into 2 prime numbers:

42 - 2 = 40 (not prime)
42 - 3 = 39 (not prime)
42 - 5 = 37 (prime) → valid
42 - 7 = 35 (not prime)
42 - 11 = 31 (prime) → valid
42 - 13 = 29 (prime) → valid
42 - 17 = 25 (not prime)
42 - 19 = 23 (prime) → valid

2nd time: split the left part into 2 numbers(the left part is an odd number, so 1 of the 2 prime numbers must be 2)

5 - 2 = 3 (prime) → valid
37 - 2 = 35 (not prime)
11 - 2 = 9 (not prime)
31 - 2 = 29 (prime) → valid
13 - 2 = 11 (prime) → valid
29 - 2 = 27 (not prime)
19 - 2 = 17 (prime) → valid
23 - 2 = 21 (not prime)

3rd time: split the left part into the sum of a prime number and a non-zero even number
3(×)
29 = 3 + 26 = 5 + 24 = 7 + 22 = 11 + 18 = 13 + 16 = 17 + 12 = 19 + 10 = 23 + 6
11 = 3 + 8 = 5 + 6 = 7 + 4
17 = 3 + 14 = 5 + 12 = 7 + 10 = 11 + 6 = 13 + 4

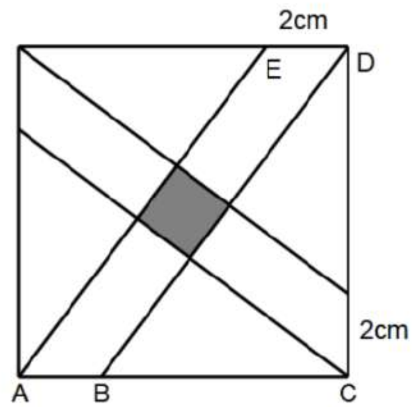
Total: 8 + 3 + 5 = 16

9. $5\times 7^3\times 35$ can be written as x^y. What is the minimum value of $x+y$? (4-mark)

A. 177
B. 247
C. 30
D. 60026
E. None of the above

【Answer】 B
【Solution】 Prime factorization
$5\times 7^3\times 35=5\times 7^3\times 5\times 7=5^2\times 7^4=\left(5\times 7^2\right)^2=245^2$
245 + 2 = 247

10. The side length of the following square is 8cm. Find the fraction of the shaded area to the entire square.



【Answer】 $\frac{1}{25}$
【Solution】 The quadrilateral ABDE is a parallelogram with an area of $2\times 8=16$
If we think DB as base, so the side of the square is the height,
In right triangle BCD, CD = 8, BC = 8 - 2 = 6, so BD = 10.
So the side is $16\div 10=1.6$
The area of the square is 1.6^2
Fraction: $\frac{1.6^2}{8^2}=0.04=\frac{1}{25}$

11. What is the number of trailing zeros for $1\times 2\times 3\times 4\times ...\times 2025$?

【**Answer**】 505

【**Solution**】 To find the number of **trailing zeros** in the product 1×2×3×··· ×2025=2025!, we need to count how many times **10** divides into 2025!.

Since 10=2×5, and there are **more 2s than 5s** in the prime factorization of 2025!, we only need to count the number of **factors of 5** in 2025!.

We can use the formula: $\left\lfloor \frac{2025}{5} \right\rfloor + \left\lfloor \frac{2025}{25} \right\rfloor + \left\lfloor \frac{2025}{125} \right\rfloor + \left\lfloor \frac{2025}{625} \right\rfloor = 405+81+16+3=505$

12. An eight-digit phone number is divisible by 99. The phone number is as follows: $\overline{85a32b63}$. Find $11a+9b$.

【**Answer**】 85

【**Solution**】 This number is divisible by **99**, so it's divisible by **both 9 and 11**, since 99 = 9 × 11, and 9 and 11 are co-prime.

We will solve this using **divisibility rules** of 9 and 11.

Step 1: Use the divisibility rule for 9

A number is divisible by **9** if the **sum of its digits** is divisible by 9.

Let's write the digits: $8+5+a+3+2+b+6+3=27+a+b$.

So $a+b=0$, 9 or 18

Step 2: Use the divisibility rule for 11

A number is divisible by 11 if the **alternating sum** of its digits is divisible by 11.

Write the digits in order:

8 5 a 3 2 b 6 3

So alternating sum is:

$8-5+a-3+2-b+6-3=(8-5-3+2+6-3)+(a-b)=5+(a-b)$.

So $5+a-b=0$,or 11

By checking the cases in Step 1 and Step 2, we find that $a=2, b=7$.

So $11a+9b=85$.

13. $\frac{3^2+5^2+7^2+...+2025^2-(2^2+4^2+6^2+...+2024^2)}{(3+5+7+...+2025)-(2+4+6+...+2024)}=?$

【**Answer**】 2027

【**Solution**】

$$\frac{3^2+5^2+7^2+...+2025^2-(2^2+4^2+6^2+...+2024^2)}{(3+5+7+...+2025)-(2+4+6+...+2024)}$$

$$=\frac{(3^2-2^2)+(5^2-4^2)+(7^2-6^2)+...+(2025^2-2024^2)}{(3-2)+(5-4)+(7-6)+...+(2025-2024)}$$

$$=\frac{(3-2)(3+2)+(5-4)(5+4)+(7-6)(7+6)+...+(2025-2024)(2025+2024)}{(3-2)+(5-4)+(7-6)+...+(2025-2024)}$$

$$=\frac{2+3+4+...+2025}{1012}=\frac{\frac{(1+2025)\times 2025}{2}-1}{1012}=2027$$

14. A, B, C, and D all have different numbers of items and know each person's item count. They said:
A: I have neither the most nor the least items
B: I do not have the least items
C: I have the most items
D: I have the least items
If exactly one of them must be lying, who is lying?

【Answer】 C

【Solution】

Try assuming each person is the liar and check for contradictions.

Case 1: Assume A is lying.
So A **does** have either the most or the least.
So suppose A has the **most**:
But then **C** says "I have the most" → then C would be lying → contradiction (we can't have two liars).
Suppose A has the **least**:
Then **D** says "I have the least" → D is lying → again two liars

A cannot be the only liar.

Case 2: Assume B is lying.
So B **does** have the **least** items.
Check D: "I have the least" → D is wrong, **B has the least**, so D is **also** lying
⇒ Two liars → contradiction
B cannot be the only liar

Case 3: Assume C is lying
So C **does not** have the most items.
Now check:
- A: "Neither most nor least" → could be true
- B: "Not the least" → okay
- D: "I have the least" → okay

Then:
- The most must be **someone other than C** — could be B or A

Let's suppose:
- D has least
- C is not most
- A is neither most nor least
- B is not least

Everything fits.
So only **C** is lying. All others' statements are consistent.
Case 4: Assume D is lying
So D **does NOT** have the least.
Then someone else must have the least → check who.
- A: "Neither most nor least" → okay
- B: "Not the least" → okay
- C: "Most" → okay

But now **no one is saying they have the least** except D, and D is lying → no one admits to least, and all others must be telling the truth. So **nobody is claiming the least**, but someone **must** have it.
Contradiction. So D cannot be the only liar.

15. $\frac{1025\times 2025}{410+205^2}+\frac{1025\times 2025}{414+207^2}+...+\frac{1025\times 2025}{446+223^2}=?$

【**Answer**】 450

【**Solution**】 $\frac{1025\times 2025}{410+205^2}+\frac{1025\times 2025}{414+207^2}+...+\frac{1025\times 2025}{446+223^2}$

$=1025\times 2025\times (\frac{1}{205\times 2+205^2}+\frac{1}{207\times 2+207^2}+...+\frac{1}{223\times 2+223^2})$

$=1025\times 2025\times (\frac{1}{205\times (2+205)}+\frac{1}{207\times (2+207)}+...+\frac{1}{223\times (2+223)})$

$=1025\times 2025\times (\frac{1}{205\times 207}+\frac{1}{207\times 209}+...+\frac{1}{223\times 225})$

$=1025\times 2025\times \frac{1}{2}\times (\frac{1}{205}-\frac{1}{207}+\frac{1}{207}-\frac{1}{209}+...+\frac{1}{223}-\frac{1}{225})$

$=1025\times 2025\times \frac{1}{2}\times (\frac{1}{205}-\frac{1}{225})$

$=\frac{1025\times 2025\times 20}{2\times 205\times 225}$

$=450$

16. 25 students took a test and got 2025 points in total. They all got distinct scores. The highest among them got 94. What is the minimum value of the lowest score? (4 mark)

【**Answer**】 45

【**Solution**】 To make the lowest score the minimum, the other score must be as greater as possible. So the other 24 scores is $71\sim 94$

$71+72+\cdots+94=(71+94)\times 24\div 2=1980$

Then the lowest score is $2025-1980=45$

17. There is a total of 100 red, blue, green and yellow marbles. The ratio of the number of blues to green marbles is 5:1. If we add 134 blue marbles the number of blues to red marbles is 9:1. The number of red marbles is a multiple of the number of green marbles. How many yellow marbles are there?

【**Answer**】 72

【**Solution**】 Set r to be x.

$b=9x-134$

$g=\frac{9x-134}{5}$

Now, we can start with trial and error. Let $x=16,21,26...$

If $x=16$, then $g=2$, $b=10$, $y=72$

If $x=21$, then $g=11$, $b=55$, $y=13$

Since r is multiple of g, so the answer is 72.

18. There are 60 red and black beads. If the beads are put into 6 boxes in lots of 10, then there will be at least one box with more than 3 red beads. If the beads are put into 3 boxes in lots of 20, then all boxes will have at least 1 black bead. How many black beads are there?

【Answer】 41

【Solution】 Consider the worst case,

For 1st situation, there are at least $3 \times 6 + 1 = 19$ red beads, so the black beads is no more than $60 - 19 = 41$

For 2nd situation, there are at least $20 \times 2 + 1 = 41$ black beads

So the number of the black beads is 41.

19. The table below shows the number of days Cindy and Diana spend to complete projects X and Y. Given that they can work individually or together, what is the least number of days needed to complete both projects?

	Cindy	Diana
X	10	7
Y	16	20

【Answer】 12

【Solution】 Let Diana finish X first, which cost 7 days,

In 7 days, Cindy finish $\frac{7}{16}$ of Y.

They need $(1 - \frac{7}{16}) \div (\frac{1}{16} + \frac{1}{20}) = 5$ more days.

$7 + 5 = 12$.

20. There is a 9-digit number that includes all digits from 1 to 9. It also has the following properties.

(a) All square numbers digits are adjacent

(b) The sum of the first 6 digits is less than the sum of the last 3 digits

(c) The first 4 digits are divisible by 16 if it is an individual number

(d) All 2 consecutive digits cannot be both even or both odd

If the number meets all the conditions, what is the remainder when it is divided by 7?

【Answer】 6

【Solution】

Given:

(a) All square digits are adjacent (Square digits: 1, 4, 9)

(b) Sum of first 6 digits < sum of last 3 digits

Since all digits 1–9 are used once, the total sum is:

$1 + 2 + 3 + \cdots + 9 = \frac{9 \times 10}{2} = 45$

Sum of first 6 digits must smaller than half of the total sum, 22.5.

Sum of last 3 digits must larger than half of the total sum, 22.5.

So sum of the last 3 digits must be **equal or larger than 23.**

Since $7 + 8 + 9 = 24$, then the last 3 digits can only be **23 or 24**

$23 = 9 + 8 + 6,\ 24 = 9 + 8 + 7$

Since 9 must be adjacent to 1 and 4, so the 3rd digit from right to left is 9

(d) All 2 consecutive digits cannot be both even or both odd
Between every two odds must be an even. We'll need to interleave them.
so the last two digit must be 8 and 7, and the left side of 9 must be 1 and 4, which means the last 5 digits is **14987**.

(c) The first 4 digits are divisible by 16 if it is an individual number

$\overline{abcd}$ (consist of 2, 3, 5 and 6) is multiple of 16,

So $\overline{cd}$ must be a multiple of 4, $\overline{cd}$ can be 32, 52, 36, 56.

Since all 2 consecutive digits cannot be both even or both odd, $\overline{abcd}$ can only be 5632, 3652, 5236, 3256. check one by one, only 5632 is a multiple of 16.
So the 9-digit number is **563214987,**
$563214987 \equiv 987 - 214 + 563 = 1336 \equiv 336 - 1 = 335 \equiv 6 \pmod 7$.