Part 1. Fill in the blanks. (8' × 4 = 32')

1. $\left|\frac{1}{3}-\frac{1}{2}\right|\times 36-\left(-2\right)^2+\left(3-\pi\right)^0$ = ___________.

【Answer】3

【Solution】the given expression $=\frac{1}{6}\times 36-4+1=3$

2. Given that $A=x^2+xy-2x-3$, $B=-x^2+3xy-9$. If $3A-B$ is -2, then the value for the expression $x^2-\frac{3}{2}x+\frac{5}{2}$ = ___________.

【Answer】2

【Solution】$3A-B=3\left(x^2+xy-2x-3\right)-\left(-x^2+3xy-9\right)=4x^2-6x$

$\because 3A-B = -2$

$\therefore 4x^2-6x = -2$

$\therefore x^2-\frac{3}{2}x = -\frac{1}{2}$

$\therefore x^2-\frac{3}{2}x+\frac{5}{2} = 2$

3. If real numbers x, y satisfy $4x+y=9$ and $2x-y=a$ with $x+y\geq 0$, then the maximum value of a = ___________.

【Answer】9

【Solution】

$\because \left(4x+y\right)-\left(2x-y\right)=2x+2y$

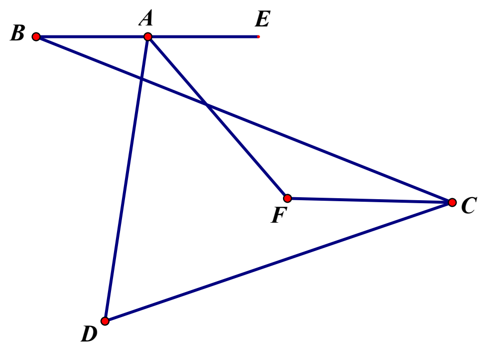
$\therefore 9-a=2x+2y$

$\because x+y\geq 0$

$\therefore 9-a\geq 0$

$\therefore a\leq 9$

4. As shown in the figure below, $\angle B = 26°$, $\angle D = 62°$, the angle bisectors of $\angle DAE$ and $\angle BCD$ cross at point F, then the degree measure of $\angle AFC$ = ________°.



【Answer】134°

【Solution】∵ $\angle B + \angle BAD = \angle BCD + \angle D$,

∴ $\angle BAD - \angle BCD = \angle D - \angle B = 36°$,

∵ $\angle BAD + \angle DAE = 180°$,

∴ $\angle DAE = 180° - \angle BAD = 144° - \angle BCD$,

Hence $\angle DAE + \angle BCD = 144°$,

Also ∵ AF and CF bisect $\angle DAE$ and $\angle BCD$ respectively,

∴ $\angle DAF = \frac{1}{2}\angle DAE$, $\angle DCF = \frac{1}{2}\angle BCD$,

∴ $\angle AFC = \angle D + \angle DAF + \angle DCF = \angle D + \frac{1}{2}(\angle DAE + \angle BCD) = 62° + \frac{1}{2} \times 144° = 134°$

Part 2. Fill in the blanks. (10' × 4 = 40')

5. If a positive integer n satisfy the condition that when 66, 78 and 42 are divided by n, the resulting remainders are all equal and nonzero, then the sum of all possible values of n is ________.

【Answer】16

【Solution】When 66, 78 and 42 are divided by n, the resulting remainders are all equal.

Hence $78 - 66 = 12$, $12 \div n$ has remainder 0

Similarly, $66 - 42 = 24$, $24 \div n$ has remainder 0

Hence, possible values of n must be factors of 12: 1, 2, 3, 4, 6, 12

Also, when 66, 78 and 42 are divided by n, the resulting remainders are nonzero.

Hence 1, 2, 3, 6 do not satisfy the conditions.

The only possible values are: 4, 12.

$4 + 12 = 16$

6. The minimum value of algebraic expression $4x^2 - 12xy + 10y^2 + 6y + 14$ is __________.

【Answer】5

【Solution】$4x^2 - 12xy + 10y^2 + 6y + 14 = (2x + 3y)^2 + (y + 3)^2 + 5 \geq 5$

7. The minimum value of $|x-1|+2|x-2|+3|x-3|+\cdots+100|x-100|$ is ________.

【Answer】99080

【Solution】

The key lies in determining the value of x. After that, the calculation still requires ensuring that the problem can be approached using the geometric property of absolute values—viewing it as the sum of distances from a point on the number line to 5050 points. (These 5050 points are: one 1, two 2's, three 3's, …, one hundred 100's.)

We look for the point in the middle, namely the 2525th and 2526th points.

Since 1+2+3+⋯+71=2556, we know the middle point is 71.

Therefore, taking x=71 makes the expression attain its minimum value.

After that, it's just a matter of carrying out the computation:

$$70+69\times 2+68\times 3+\ldots+1\times 70+0\times 71+1\times 72+2\times 73+\ldots+29\times 100$$

$$=(71-1)+(71\times 2-2\times 2)+(71\times 3-3\times 3)+\ldots+(71\times 70-70\times 70)+0+(1\times 71+1)+(2\times 71+2\times 2)+\ldots+(29\times 71+29\times 29)$$

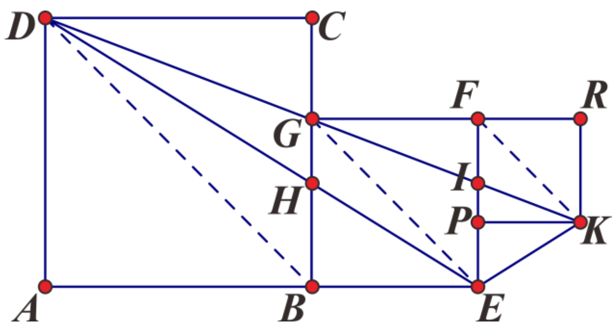
$$=71\times(1+2+3+\ldots+70)-(1^2+2^2+3^2+\ldots 70^2)+71\times(1+2+3+\ldots+29)+(1^2+2^2+3^2+\ldots 29^2)$$

$$=71\times\frac{1}{2}\times 70\times 71-\frac{1}{6}\times 70\times 71\times 141+71\times\frac{1}{2}\times 29\times 30+\frac{1}{6}\times 29\times 30\times 59$$

$$=99080$$

8. Three squares $ABCD$, $BEFG$, $PKRF$ are placed as shown below, point G is on the line segment DK, and square $BEFG$ has a side length of 2. Then the area of $\triangle DEK$ is ________.

【Answer】4



【Solution】$\because DB // GE // FK$

$\therefore S_{\triangle DGH}=S_{\triangle BHE}$

$S_{\triangle IKE}=S_{\triangle GFI}$

$\therefore S_{\triangle DEK}=4$

Part 3. Fill in the blanks. (14' × 3 = 42')

9. From a convex n-gon (a polygon with n sides), several non-adjacent interior angles are picked, and the maximum possible value for the sum of these interior angles is 600°, then the sum of all the possible values of n is ________.

【Answer】17

【Solution】If n=7, in a heptagon at most 3 non-adjacent interior angles can be picked.

Since in a convex polygon, each interior angle is less than 180°,

$180°\times3=540°<600°$, n=7 does not meet the conditions.

We have $n\geqslant8$.

If n=10, consider a regular decagon, where each interior angle is 144° ($180°\times8\div10=144°$), then at most 5 non-adjacent interior angles can be picked.

$144°\times5=720°>600°$, n=10 does not meet the conditions.

We have $n\leqslant9$

Hence n = 8 or 9.

Double check to find if n=8, n=9, such polygons do exist.

Hence answer is:

8+9=17

10. A 3-digit integer is written as $\overline{abc}$, where a,b,c are three distinct numbers. When $\overline{abc}$ is divided by 2-digit integers $\overline{bc},\overline{ac}$ respectively, the resulting remainders are both a, then the sum of all the possible $\overline{abc}$ is ____________.

【Answer】618

【Solution】$100a+10b+c\equiv a\left(\bmod \overline{bc}\right)$, $\left(10b+c\right)|99a$,

Since $b\neq c$, we have $\left(10b+c,11\right)=1$ (these two numbers are coprime)

Hence $\left(10b+c\right)|9a$

And since a,b cannot be 0, by enumeration method, we may check when $a=1,2,3,\cdots,9$, what are the possible 2-digit multiples of $9a$.

We will find the possible values for the 3-digit integer $\overline{abc}$ are:

$\overline{abc}=218,327,436,418,412,618,627,654,721,872,836,824,812,827$

Then we check $\overline{abc}\equiv a\left(\bmod \overline{ac}\right)$, only 618 satisfies this condition.

11. Two table tennis players, A and B, are competing. The current score is tied at 5-5. It is known that throughout the entire match, player B has only led player A once. Then the number of different possible score progressions is ____________.

【Answer】42

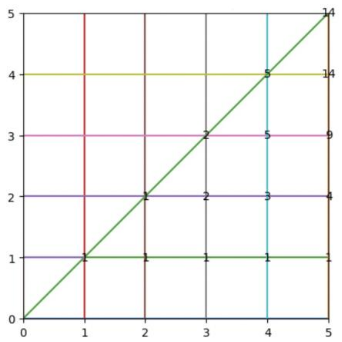
【Solution】

Mapped onto the Cartesian coordinate plane, the path representing the score starts from the origin $(0,0)$ and moves to the endpoint $(5,5)$. Let's define that when player A scores 1 point, we move rightwards by 1, and when player B scores 1 point, we move upwards by 1. Since player B only led once, the path can cross above the line $y=x$ only once - that is, it can touch exactly one of the five points: $(0,1)(1,2)(2,3)(3,4)(4,5)$ and then immediately return to $y=x$ or below this line. So, we consider these 5 points case by case using labeling method.

Case 1: when the path passes (0,1).

Before this point, there is only 1 way to reach (0,1).

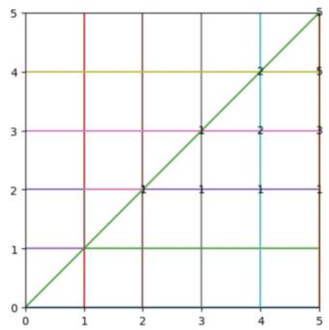
After this point, by labeling method, there are 14 distinct paths.



Case 2: when the path passes (1,2).

Before this point, there is only 1 way to reach (1,2).

After this point, by labeling method, there are 5 distinct paths.



Case 3: when the path passes (2,3).

Before this point, there are 2 ways to reach (2,3).

After this point, by labeling method, there are 2 distinct paths.

In general, there are 4 paths.

Case 4: when the path passes (3,4).

It is “symmetric” to case 2 and there are 5 paths.

Case 5: when the path passes (4,5).

It is “symmetric” to case 1 and there are 14 paths.

In all, we can find the total number of distinct score progressions: $14+5+4+5+14=42$.

Part 4. Answer the following questions. Show all steps clearly. (18' × 2 = 36')

12. A real number $a>0$, b is the decimal part of a, and $a^2+b^2=2024$, find the value of a^3-b^3.

【Answer】90992

【Solution】$\because$ b is the decimal part of a

$\therefore 0\le b^2<1$,

$\because a^2+b^2=2024$,

$\therefore 2023<a^2\le 2024$,

$\because 45^2=2025$, $44^2=1936$,

$\therefore$ The integer part of a is 44, that is, $a-b=44$,

$\therefore (a-b)^2=a^2+b^2-2ab=2024-2ab=44^2=1936$, we have $ab=44$,

$\therefore a^3-b^3=(a-b)\left(a^2+ab+b^2\right)=44\times(2024+44)=90992$.

13. From $1,2,3,\cdots,1000$, n numbers are picked. These n numbers can be arranged in a row such that the product of every three adjacent numbers does not exceed 2000. Prove: $n\le 34$.

【Answer】34

【Solution】(1) Since $12\times 13\times 14=2184>2000$,

then in "every three adjacent number", there must be at least one number from $1,2,3,\cdots,11$;

(2) If $n\ge 36$, then we need at least 12 distinct numbers from $1,2,3,\cdots,11$, which is a contradiction;

(3) If $n=35$, then we can only place $1,2,3,\cdots,11$ at the $3rd, 6th, 9th, \cdots, 33th$ position. Consider 11, we need to construct __, __, 11, __, __ such that the product of the LHS 3 numbers and the product of the RHS 3 numbers both do not exceed 2000. Since $11\times 13\times 14=2002>2000$, then 12 is needed on both sides of 11, which is a contradiction;

Therefore, $n\le 34$.