数学花园
探秘
MATHEMATICS GARDEN

2026 Spring Cup
Mathematical Olympiad

Date: 28 February 2026

Time Given: 1 hour 30 minutes

Level: Primary 5

Name: ____________

Instructions to Candidates

1. Do not open the booklet until you are told to do so.
2. Answer ALL 20 questions.
3. Write your answers in the answer sheet provided.
4. No steps are needed to justify your answers.
5. Questions 1-7 are worth 4 marks each.
6. Questions 8-14 are worth 6 marks each.
7. Questions 15-19 are worth 8 marks each.
8. Question 20 is worth 10 marks.
9. No marks will be deducted for wrong answers.
10. No marks will be given for unanswered questions.
11. No calculators or mathematical instruments are allowed.

Questions 1 to 7 are worth 4 marks each.

1. Find the value of $76\times(\frac{1}{23}-\frac{1}{53})+23\times(\frac{1}{53}+\frac{1}{76})-53\times(\frac{1}{23}-\frac{1}{76})$.

【Answer】1

【Solution】

$=76\times\frac{1}{23}-76\times\frac{1}{53}+23\times\frac{1}{53}+23\times\frac{1}{76}-53\times\frac{1}{23}+53\times\frac{1}{76}$

$=76\times\frac{1}{23}-53\times\frac{1}{23}-76\times\frac{1}{53}+23\times\frac{1}{53}+23\times\frac{1}{76}+53\times\frac{1}{76}$

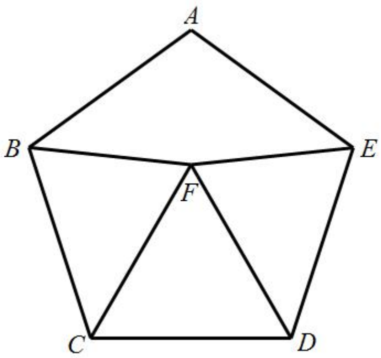
$=\frac{1}{23}\times(76-53)-\frac{1}{53}\times(76-23)+\frac{1}{76}\times(23+53)$

$=\frac{1}{23}\times(23)-\frac{1}{53}\times(53)+\frac{1}{76}\times(76)$

$=1-1+1$

$=1$

2. As shown, the pentagon *ABCDE* is a regular pentagon and the triangle *CDF* is an equilateral triangle. Find $\angle BFE$ (less than 180 degrees)?



【Answer】168°

【Solution】

The interior angle for a regular pentagon is $\frac{(5-2)\times180°}{5}=108°$

Since triangle *CDF* is an equilateral triangle, we know that it has an interior angle of 60° and *CF* = *FD* = *CD*. Since pentagon *ABCDE* is a regular pentagon, we know that *CD* = *BC* = *ED* which implies that *FD* = *ED* and *CF* = *BC*. Hence, triangle *BCF* and *EFD* are isosceles triangle.

Therefore, $\angle EDF=108°-60°=48°$ and $\angle EFD=\frac{180°-48°}{2}=66°$.

$\angle BFE=360°-60°-66°\times2=168°$

3. In a football round-robin tournament involving four teams—A, B, C, and D—each pair of teams plays exactly 1 match. A win earns 2 points, a loss earns 0 points, and a draw earns 1 point for each team. Teams A, B, and C currently have 5 points, 1 point, and 4 points respectively. Given that Team A ties with Team B, what is Team D's score?

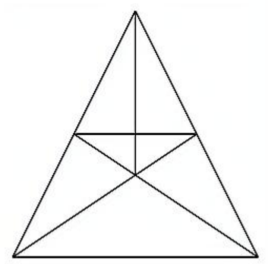
【Answer】2

【Solution】

This is a round-robin tournament, they play against each other for only 1 match. A have 5 points which means it must get 2 wins and 1 draw. B has 1 point which means it must get 2 loses and 1 draw. C has 4 points which means it either have 2 wins and 1 lose or 1 win and 2 draw. Since Team A ties with Team B, we know Team A win the other 2 teams. Therefore, Team C must have 2 wins and 1 lose. Table below shows the information that we get from the questions. Team D’s score is 2.

Against Team	A	B	C	D
A	-	1	2	2
B	1	-	0	0
C	0	2	-	2
D	0	2	0	-

4. Find the number of triangles of the figure.



【Answer】20

【Solution】

Counting from the smallest to the largest
Triangles made of 1 part: 7
Triangles made of 2 parts: 6
Triangles made of 3 parts: 4
Triangles made of 5 parts: 2
Triangles made of 7 parts: 1
7 + 6 + 4 + 2 + 1 = 20 triangles

5. In a class of 30 students, the average score for a Science test is 68. Two students, Daniel and Ethan, leave the class. The average score of the remaining students increases to 69. Given that Daniel scores 4 more marks than Ethan, what is Daniel's score?

【Answer】56

【Solution】

Total score of 30 students is $68 \times 30 = 2040$
Total score after Daniel and Ethan leave is $69 \times 28 = 1932$
Total score of Daniel and Ethan is $2040 - 1932 = 108$
Since Daniel scores 4 more marks than Ethan, Daniel marks is $(108-4) \div 2 + 4 = 56$

Or

When Daniel and Ethan leave, the 28 remaining students see their average rise from 68 to 69. This means those 28 students no longer have to share 1 mark each with Daniel and Ethan.
Total marks given to Daniel and Ethan is $28 \times 1 = 28$ marks.
$28 \div 2 = 14$ marks are given to each of them to make average 68 which implies that the average of Daniel and Ethan is $68 - 14 = 54$.
Total score of Daniel and Ethan is $54 \times 2 = 108$.
Since Daniel scores 4 more marks than Ethan, Daniel marks is $(108-4) \div 2 + 4 = 56$.

6. A car travels from point A to point B at 100 km/h, then returns along the same route at 60 km/h. Find the car's average speed for the whole trip.

【Answer】75 km/h

【Solution】

We know the speed to complete the same route but the distance is unknown. Hence, we try to set a specific distance for the route. To make calculations easy, choose a distance that is a common multiple of 100 and 60, such as 300 km.
Time from A to B is $300 \div 100 = 3$ hours.
Time from B to A is $300 \div 60 = 5$ hours.
Total time is 3 + 5 = 8 hours.
Total distance is 300 + 300 = 600 km.
Average speed is $600 \div 8 = 75$ km/h

7. There is a 2-digit number x. The remainder when $(x+6)$ is divided by 3 is 1, the remainder when $(x-8)$ is divided by 4 is 1. Find the maximum possible value for x.

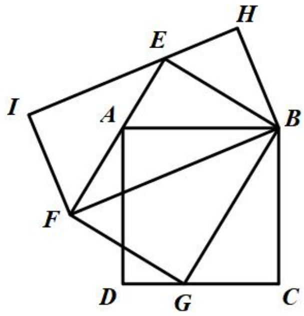
【Answer】97

【Solution】

Since 6 is divisible by 3, this is the same as saying $x \div 3$ leaves a remainder of 1.
Since 8 is divisible by 4, this is the same as saying $x \div 4$ leaves a remainder of 1.
$(x-1)$ must be a number that can be divided by both 3 and 4. This means $(x-1)$ is a multiple of the Least Common Multiple (LCM) of 3 and 4, LCM(3, 4) = 12.
Since we want x to be the maximum possible value and it is a 2-digit number.
The largest multiple of 12 that is a 2-digit number will be 96.
$(x-1)$ is 96 and we get the maximum possible value $x = 97$.

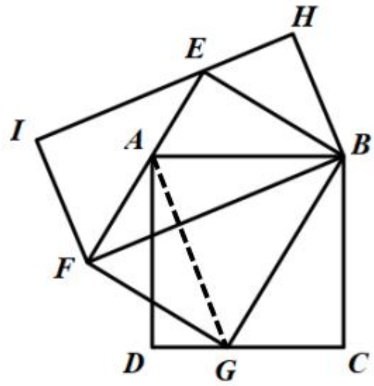
<u>Questions 8 to 14 are worth 6 marks each.</u>

8. As shown in the figure, the square *ABCD* has a side length of 8 cm, and the rectangle *BFIH* has a length of 10 cm. What is the width of the rectangle?



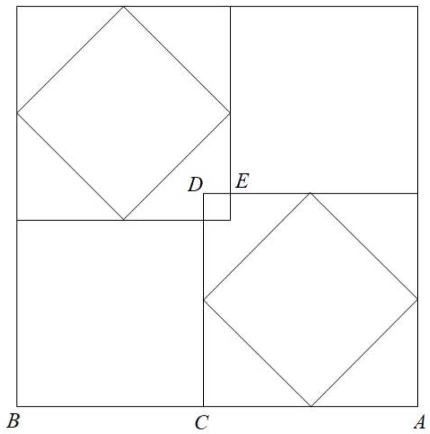
【Answer】6.4 cm

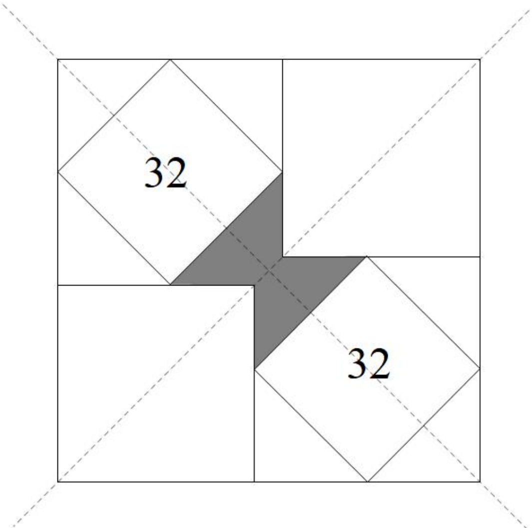
【Solution】



Connect line *AG*. The area of the square *ABCD* is $8 \times 8 = 64$ cm^2. Based on the Half Model, we can observe that triangle *ABG* is half of the square *ABCD* with an area of $64 \div 2 = 32$ cm^2. Based on the Half Model, we can observe that triangle *ABG* is also half of the rectangle *EBGF*. Based on the Half Model, we can observe that triangle *EBF* is half of the rectangle *EBGF* and *BFIH*. This means that the area of triangle *EBF* = area of triangle *ABG* = 32 cm^2 and the area of rectangle *BFIH* is $32 \times 2 = 64$ cm^2. Hence, the width of the rectangle *BFIH* is $64 \div 10 = 6.4$ cm.

9. As shown in the figure, in a square with an area of 225, place four small squares to form a symmetrical figure with two lines of symmetry. Two squares with an area of 32 are marked and the other 2 square are also the same. Find the area of the shaded region.





【Answer】15

【Solution】

According to the question, *AB* = 15, *AC* = 8, so *BC* = 15 – 8 = 7, *DE* = 8 – 7 = 1. The shaded area is covered by two overlapping isosceles right triangles, and its area is 32 ÷ 2 – 1 = 15.

10. Use the 2026 numbers 2, 3, 4,..., 2026,2027 as numerators and denominators to construct 1013 fractions. The minimum value of the largest fraction among these 1013 fractions is $\frac{b}{a}$. Find the value of $a+b$.

【Answer】3041

【Solution】

(1) Let the larger half of the numbers as the denominator and the smaller half of the numbers as the numerator. Arrange in an increasing order: $\frac{2}{1015}$, $\frac{3}{1016}$, …, $\frac{1014}{2027}$. Based on concentration principle, the largest fraction is $\frac{1014}{2027}$.

(2) If we want to find a largest fraction that is smaller than $\frac{1014}{2027}$, then 2027 cannot be the numerator and it needs to pair with a numerator smaller than 1014. Considering 1014, 1015, …, 2026, these 1013 numbers will have 1 number that must be a numerator and pair with a denominator smaller than 2027. This result to a fraction that is larger than $\frac{1014}{2027}$.

Hence, the minimum value of the largest fraction is $\frac{1014}{2027}$.

the sum of $a+b=1014+2027=3041$

11. From City P to City Q, it takes Car A 3 hours and Car B 7 hours. Two cars travel towards each other from two cities and meet at a point 21 km away from the midpoint of PQ. What is the distance between City P and City Q in km?

【Answer】105 km

【Solution】

Assume the total distance between City P and City Q is 21 units
Speed of Car A is $21 \div 3 = 7$ units/hour
Speed of Car B is $21 \div 7 = 3$ units/hour
When two cars travel towards each other, in every hour, the total distance covered together is $7+3=10$ units.
Since the whole journey is 21 units, the time they talk to meet is $21 \div 10 = 2.1$ hours.
Distance Car A travels are $2.1 \times 7 = 14.7$ units.
Distance Car A travels are $2.1 \times 3 = 6.3$ units.
The midpoint of the journey is $21 \div 2 = 10.5$ units.
Car A met Car B at 14.7 units. The distance from the midpoint is $14.7 - 10.5 = 4.2$ units.
4.2 units = 21 km and 1 unit is 5 km.
Total distance is $21 \times 5 = 105$ km.

12. Alex write a sequence with 2 pattern as below, he can write any one pattern randomly, but he must finish writing before modifying the next pattern.

(i) AAABAAC

(ii) AABBACC

After he finished a whole number of patterns, he found there are 125 "A"s and 46 "B"s. Find the number of complete pattern of (i).

【Answer】16

【Solution】

From the pattern (i) and (ii), we can find that the number of B and C are equal in each pattern.
So there must be also 46 "C"s.
There are $125+46+46=217$ letters in total.
So $217\div 7=31$ groups
If all the 31 groups are Pattern (i), there will be 31 "B"s.
There are $46-31=15$ letters "B"s more than 31.
Changing each pattern (i) to (ii), there will be 1 letter "B" added.
So we need $15\div 1=15$ groups (ii).
There will be $31-15=16$ groups of pattern (i).

13. Given that $N=2^a\times 3^b$, the number of its factors is 12, and the sum of all factors is 1092. Find the sum of *a* and *b*.

【Answer】6

【Solution】

The number of factors for a number with prime factorization $p_1^a\times p_2^b$ is given by the formula $(a+1)\times(b+1)$ where p_1 and p_2 are the prime factors. Since the total number of factors is 12, we look for pairs of integers that multiply to 12: 1×12, 2×6 and 3×4.
If it is 1×12, then $a=0$, $b=11$ or $a=11$, $b=0$.
If it is 2×6, then $a=1$, $b=5$ or $a=5$, $b=1$.
If it is 3×4, then $a=2$, $b=3$ or $a=3$, $b=2$.
The formula for the sum of factors is $\left(2^0+2^1+\cdots+2^a\right)\times\left(3^0+3^1+\cdots+3^b\right)$ as we already given with the prime factors are 2 and 3.
As per checking, we can see that when $a=1$, $b=5$.
The sum of factor is

$$\begin{aligned}\left(2^0+2^1\right)\times\left(3^0+3^1+3^2+3^3+3^4+3^5\right)&=(1+2)\times(1+3+9+27+81+243)\\&=3\times 364\\&=1092\end{aligned}$$

Hence, $a=1$ and $b=5$, the sum of *a* and *b* is $1+5=6$.

14. As shown in the figure below, A~G represent the digit 1~7, the same letter represent the same digit, different letter represent different digits. Find the 4-digit number "$\overline{ABCD}$".

$$\begin{array}{cccc} & & A & B \\ & \times & C & D \\ \hline & F & C & D \\ E & A & E & \\ \hline E & G & A & D \end{array}$$

【Answer】4735

【Solution】

The puzzle contains 7 different characters, meaning the digits 1 through 7 each appear exactly once. By analyzing the positions of the letters A, B, C, D, and E, we can determine that none of them can be 1, and A must be at least 3.

The digit D can only be 2, 4, or 5.

If D were 2 or 4, then B would be 6, but the first partial product $\overline{FCD}$ would have no solution that fits the requirements. Therefore, D must be 5.

If F were 1, then A would be 3. However, this would force B to be 7, which does not satisfy the requirements.

Thus, E must be 1.

With E = 1, the digits for C and D are 3 and 7. Since A must be at least 3, we can deduce the following: C = 3 and B = 7

By following the multiplication through, we find A = 4.

By substituting these values, we find that the 4-digit number $\overline{ABCD}$ is 4735.

<u>Questions 15 to 19 are worth 8 marks each.</u>

15. Write the squares of 1~100 together to form a number $1491625\cdots980110000$. Find the remainder when $1491625\cdots980110000$ is divided by 9.

【Answer】4

【Solution】

The divisibility rule for 9 states that a number and the sum of its digits (or the sum of all part where any digit can be one part) leave the same remainder when divided by 9. Since the large number is formed by concatenating the squares 1^2, 2^2, 3^2, ..., 100^2, the remainder of the entire sequence is equivalent to the remainder of the sum:

$S = 1^2 + 2^2 + 3^2 + ... + 100^2$

To calculate the sum, we use the formula for the sum of squares: $\dfrac{n(n+1)(2n+1)}{6}$

$\dfrac{100(100+1)(201+1)}{6} = 338350$

Finally, the remainder of the sum when divided by 9 can be calculated by adding its digits.

$3+3+8+3+5+0=22$

$2+2=4$

The remainder when the number divided by 9 is 4.

16. A student is conducting a controlled experiment to investigate the relationship between auxin concentration and plant growth. He plans to prepare 2026 grams of 10%,20%,30%,40%, and 50% auxin solutions using 60% auxin and distilled water. Find the total amount of distilled water required for preparation(in gram).

【Answer】5065

【Solution】

The total amount of solution is (5×2026) grams, the concentration of the auxin solution is $\frac{10\%+20\%+30\%+40\%+50\%}{5}=30\%$.

Based on the cross method,

60% ＼ ／ 0%
30%
30% ／ ＼ 30%

.

Therefore, the ratio of 60% auxin and distilled water is 1:1.

The amount of distilled water required is $(5\times 2026)\times\frac{1}{1+1}=5065$ grams.

17. There are 12 lamps, each with one of three colors: red, yellow, or green. Allen, Benjamin, Cindy, and Daniel don't know the actual lighting status. The table below summarizes their color guesses for each lamp and the number of correct guesses:

number	1	2	3	4	5	6	7	8	9	10	11	12	correct guesses
Allen	Y	R	Y	G	R	R	G	Y	G	G	Y	R	6
Benjamin	Y	G	G	R	G	Y	Y	G	R	Y	R	Y	7
Cindy	R	G	Y	R	G	G	Y	Y	R	R	Y	Y	9
Daniel	G	Y	R	R	Y	R	R	G	R	Y	G	G	

Y - Yellow, R - Red, G - Green

Find the number of correct guesses of Daniel.

【Answer】3

【Solution】

Benjamin and Cindy gave identical guesses for lamps 2, 4, 5, 7, 9, and 12.
(1) If 5 out of 6 common answers is correct,
Since Allen's answers for these 6 lamps are all different from Benjamin's and Cindy's, this means that Allen has at least 5 wrong answers from lamps 2, 4, 5, 7, 9, and 12.

As for the remaining lamps 1, 3, 6, 8, 10, 11, Benjamin guessed $7-5=2$ correctly; Cindy guessed $9-5=4$ correctly. This means that either Benjamin or Cindy is correct.

Since Allen's answers for lamps 6 and 10 are different from both Benjamin's and Cindy's answers, this means Allen guessed lamps 6 and 10 wrongly.

Altogether, Allen has at least $5+2=7$ wrong guesses, which is impossible because he has 6 correct guesses.

(2) This means that these 6 lamps must all be correct for both of them.

Allen's guesses for those same six lamps (2, 4, 5, 7, 9, and 12) are all different from the correct colors identified above. This means Allen got 0 correct in that group.

Since Allen has a total of 6 correct guesses, he must have guessed all the other lamps (1, 3, 6, 8, 10, and 11) correctly.
By combining these findings, the actual colors of lamps 1-12 are: Yellow, Green, Yellow, Red, Green, Red, Yellow, Yellow, Red, Green, Yellow, Yellow.

Comparing Daniel's guesses to this sequence:

Lamp 4 (Red): Correct

Lamp 6 (Red): Correct

Lamp 9 (Red): Correct

Daniel made 3 correct guesses.

18. How many ways are there to colour the 2×5 grid below using 4 different colours such that each cell is coloured with only one colour and the colour of adjacent cells must be different.

【Answer】28812

【Solution】

A	B
D	C

Separate into two cases based on whether opposite regions share the same colour.
When we have 2×2 grid, we need to consider whether A = C or A ≠ C. Let's try to find number of ways by following the order of the letter.
When A ≠ C, the number of ways to colour the grid is $4\times3\times2\times2$ because A have 4 choices then B have 3 choices, since A and C is not the same, C will have 2 choice left and D also 2 choices.
When A = C, the number of ways to colour the grid is $4\times3\times1\times3$ because A have 4 choices then B have 3 choices, since A and C is the same, C will only have 1 choice which is to follow A and D will have 3 choices.
So our final calculation is

$$\begin{aligned}4\times3\times2\times2+4\times3\times1\times3 &= (4\times3)\times(2\times2+1\times3)\\ &=12\times7\end{aligned}$$

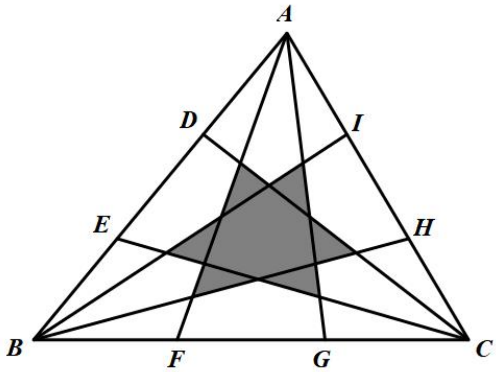
A	B	E
D	C	F

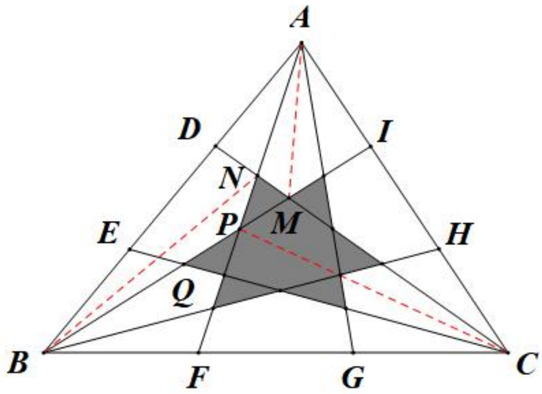
When we have 2×3 grid, we need to consider whether B = F or B ≠ F. Let's try to find number of ways by following the order of the letter.
When B ≠ E, the number of ways to colour the grid is $12\times7\times2\times2$.
When A = C, the number of ways to colour the grid is $12\times7\times3\times1$.
So our final calculation is

$$\begin{aligned}12\times7\times2\times2+12\times7\times7\times1 &= 12\times7\times(2\times2+1\times3)\\ &=12\times7\times7\end{aligned}$$

Based on the example, we can see that for every column added to the grid, we just have to multiply by another 7.
Hence, the number of ways to colour the 2×5 grid is $12\times7\times7\times7\times7=28812$.

19. As shown in the figure, triangle ABC has an area of 140. $AD = DE = EB$, $BF = FG = GC$, $CH = HI = IA$, find the area of the shaded part.





【Answer】26

【Solution】

Let M be the intersection of BI and CD, N the intersection of AF and CD, P the intersection of BI and AF, and Q the intersection of BI and CE. Connect AM, BN, and CP.

(1) Find the area of ADMI

In $\triangle ABC$, according to the swallowtail model,

$S_{\triangle ABM}:S_{\triangle CBM}=AI:CI=1:2$, $S_{\triangle ACM}:S_{\triangle CBM}=AD:BD=1:2$

Let $S_{\triangle ABM}=1$ u, $S_{\triangle CBM}=2$ u, $S_{\triangle ACM}=1$ u, $S_{\triangle ABC}=4$ u.

So $S_{\triangle ABM}=S_{\triangle ACM}=\frac{1}{4}S_{\triangle ABC}$, $S_{\triangle ADM}=\frac{1}{3}S_{\triangle ABM}=\frac{1}{12}S_{\triangle ABC}$, $S_{\triangle AIM}=\frac{1}{12}S_{\triangle ABC}$,

Then $S_{ADMI}=(\frac{1}{12}+\frac{1}{12})S_{\triangle ABC}=\frac{1}{6}S_{\triangle ABC}$,

Similarly, the area of the other 2 quadrilateral formed by the other two vertices are also $\frac{1}{6}$ of $\triangle ABC$.

(2) Find the area of pentagon $DNPQE$

In $\triangle ABC$, according to the swallowtail model,

$S_{\triangle ABN}:S_{\triangle ACN}=BF:CF=1:2$, $S_{\triangle ACN}:S_{\triangle BCN}=AD:BD=1:2$

So $S_{\triangle ADN}=\frac{1}{3}S_{\triangle ABN}=\frac{1}{3}\times\frac{1}{7}S_{\triangle ABC}=\frac{1}{21}S_{\triangle ABC}$

Similarly, $S_{\triangle BEQ}=\frac{1}{21}S_{\triangle ABC}$

$S_{\triangle ABP}:S_{\triangle ACP}=BF:CF=1:2$, $S_{\triangle ABP}:S_{\triangle CBP}=AI:CI=1:2$

So $S_{\triangle ABP}=\frac{1}{5}S_{\triangle ABC}$.

$$S_{DNPQE}=S_{\triangle ABP}-S_{\triangle ADN}-S_{\triangle BEP}=\left(\frac{1}{5}-\frac{1}{21}-\frac{1}{21}\right)S_{\triangle ABC}=\frac{11}{105}S_{\triangle ABC}$$

Similarly, the area of the other 2 pentagon are also $\frac{11}{105}$ of $\triangle ABC$.

Therefore, $S_{shaded}=\left(1-\frac{1}{6}\times3-\frac{11}{105}\times3\right)\times140=26$.