2024 Spring Cup
Mathematical Olympiad
PRELIMINARY ROUND

Date: 28 January 2024

Time Given: 1 hour

Level: Primary 4&5

Name: _____

Instructions to Candidates

1. Do not open the booklet until you are told to do so.
2. Answer ALL 20 questions.
3. Write your answers in the answer sheet provided.
4. No steps are needed to justify your answers.
5. Questions 1-7 are worth 4 marks each.
6. Questions 8-14 are worth 6 marks each.
7. Questions 15-19 are worth 8 marks each.
8. Question 20 is worth 10 marks.
9. No marks will be deducted for wrong answers.
10. No marks will be given for unanswered questions.
11. No calculators or mathematical instruments are allowed.

Questions 1 to 7 are worth 4 marks each.

1. $2024\times\left(\frac{1}{11}+\frac{2}{23}\right)=$

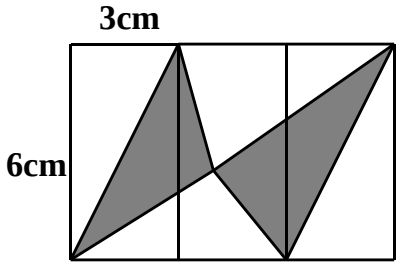
2. The sum of three whole numbers *A*, *B* and *C* is 138. Suppose that *A* is 20% more than *B* while *B* is 20% less than *C*. Find the value of *C*.

3. It is known that the whole number in each box below is the same.

$$\square\times\square+\square=870$$

What is the whole number?

4. In the figure below, three 3 cm × 6 cm rectangles are placed together in a row. Find the area (in cm^2) of the unshaded region.



5. The sum of the father and son's ages is 45 years. 5 years from now, father's age will be exactly 4 times that of the son. What is the current age of father?

6. There are 100 numbers in a row, as shown:

$$a_1, a_2, a_3, \cdots, a_{100}$$

Given that the average of these 100 numbers is 95, while the average of the first 51 numbers is 100 and the last 50 numbers is 90. Find the value of a_{51}.

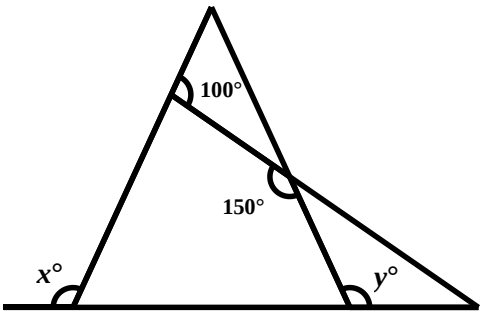
7. A train with a length of 200 meters passes through a tunnel. The time between the train head to enter and the train tail to exit the tunnel is 60 seconds. The time for the train to be completely inside the tunnel is 40 seconds. What is the speed of the train?

Questions 8 to 14 are worth 6 marks each.

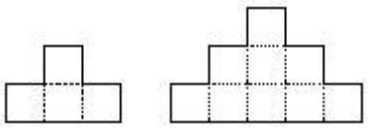
8. Benjamin drove a car from city *A* to city *B* with constant speed. If he drove 10% faster than this speed, he will reach city *B* twenty minutes earlier. How long, in minutes, did Benjamin spend for his journey, using his initial speed?

9. A glass with water weighs 172 grams when it is one-third full and 192 grams when it is half full. What is the weight, in grams, of a full glass of water (including the glass)?

10. In the figure below, find the value of $x + y$.



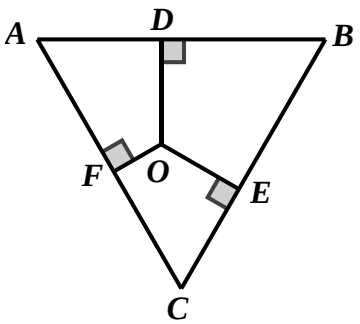
11. The two figures in the diagram (solid lines) are each surrounded by 10 and 16 sticks, respectively. If following this pattern (each layer has two additional small squares compared to the layer above) a figure surrounded by 64 sticks is formed, how many layers does the figure have?



12. How many four-digit numbers, where the thousands digit is 8 and each number has exactly two identical digits (e.g., 8336, 8545, 8782), are there?

13. In a bag, there are black and white balls. After removing 15 white balls, black balls constitute $\frac{2}{3}$ of the total. After removing 45 black balls, black balls constitute $\frac{1}{6}$ of the total. Find the initial number of black and white balls in total.

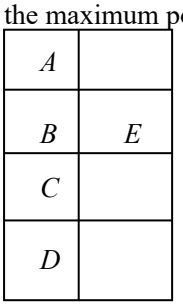
14. The diagram shows an equilateral triangle ABC with OD, OE and OF perpendicular to AB, BC and CA respectively. If OD+OE+OF=28cm and the height of triangle ABC = x cm, find the value of x.



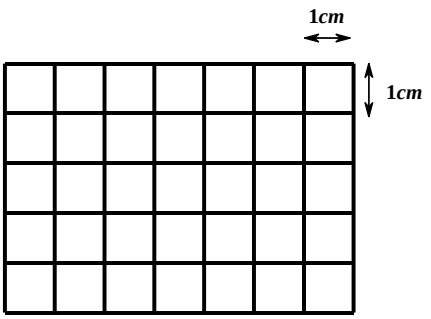
Questions 15 to 19 are worth 8 marks each.

15. What is the average of all 4-digit numbers that can be formed by using each of the digits 2, 4, 6 and 8 exactly once?

16. Given the figure, a rectangle is divided into 8 smaller rectangles, with the perimeters of rectangles A, B, C, D, and E being 26 cm, 28 cm, 30 cm, 32 cm, and 34 cm respectively. What is the maximum possible area of the largest rectangle in square centimeters?

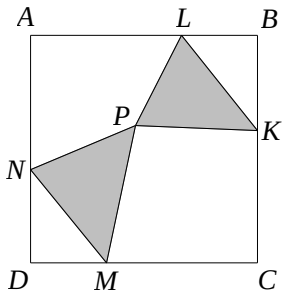


17. The figure below shows a 7×5 grid. Suppose that the side length of each small square is 1 cm. What is the sum of the areas of all rectangles with a perimeter of 12? (The square is also a rectangle.)



18. Abel, Ben and Calvin completed a project together in 13 days. For the first 6 days, Abel and Ben completed $\frac{1}{3}$ of the project together. For the next 2 days, Ben and Calvin completed $\frac{1}{6}$ of the project together. For the last 5 days, Abel, Ben and Calvin completed the rest of the project together. Ben was paid \$1820, which was distributed among them according to the amount of work they did in this project. How much money should be given to Abel?

19. *ABCD* is a square with a side length of 12, as shown in the diagram. P is a random point inside, and $BL = DM = 4$、$BK = DN = 5$. Find the area of the shaded region.



Question 20 is worth 10 marks.

20. In your opinion, from question 1 to 19, your favourite question is question _____ and the most difficult question is question _____.
(As long as your answer is within 1 to 19, you get full marks, otherwise you get zero.)