数学花园
探秘
MATHEMATICS GARDEN

2026 Spring Cup
Mathematical Olympiad

Date: 28 Feb 2026

Time Given: 1 hour 30 minutes

Level: Primary 6

Name: _____

Instructions to Candidates

1. Do not open the booklet until you are told to do so.
2. Answer ALL 20 questions.
3. Write your answers in the answer sheet provided.
4. No steps are needed to justify your answers.
5. Questions 1-7 are worth 4 marks each.
6. Questions 8-14 are worth 6 marks each.
7. Questions 15-19 are worth 8 marks each.
8. Question 20 is worth 10 marks.
9. No marks will be deducted for wrong answers.
10. No marks will be given for unanswered questions.
11. No calculators or mathematical instruments are allowed.

Questions 1 to 7 are worth 4 marks each.

1. Find the value of $(2026 + 202.6 + 20.26 + 2.026) \div 1.01 \div 1.1$.

【Answer】2026

【Solution】

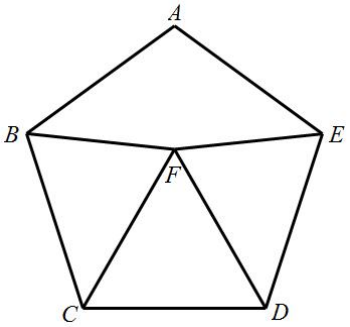
$= 2026 \times (1 + 0.1 + 0.01 + 0.001) \div 1.01 \div 1.1$

$= 2026 \times 1.111 \div (1.01 \times 1.1)$

$= 2026 \times 1.111 \div 1.111$

$= 2026$

2. As shown, the pentagon *ABCDE* is a regular pentagon and the triangle *CDF* is an equilateral triangle. Find $\angle BFE$ (less than 180 degrees)?



【Answer】168°

【Solution】

The interior angle for a regular pentagon is $\dfrac{(5-2)\times 180°}{5} = 108°$

Since triangle *CDF* is an equilateral triangle, we know that it has an interior angle of 60° and *CF* = *FD* = *CD*. Since pentagon *ABCDE* is a regular pentagon, we know that *CD* = *BC* = *ED* which implies that *FD* = *ED* and *CF* = *BC*. Hence, triangle *BCF* and *EFD* are isosceles triangle.

Therefore, $\angle EDF = 108° - 60° = 48°$ and $\angle EFD = \dfrac{180° - 48°}{2} = 66°$.

$\angle BFE = 360° - 60° - 66° \times 2 = 168°$

3. If there are four Mondays and four Thursdays in January of a given leap year, what day is June 1?

【Answer】Wednesday

【Solution】

January has 31 days. In any 31-day month, three consecutive weekdays occur 5 times and the remaining four weekdays occur 4 times. Since January has exactly four Mondays and four Thursdays, Monday and Thursday must be among the weekdays that occur only 4 times. The only three consecutive weekdays that exclude both Monday and Thursday are Friday, Saturday and Sunday. Hence, January 1 must be a Friday (so that Friday, Saturday and Sunday each occur 5 times).

Now count forward to June 1 in a leap year. The total number of days from January 1 to June 1 is $31 + 29 + 31 + 30 + 31 + 1 = 153$.

$153 \div 7 = 21$ R6

Since Friday is R1, count 6 days forward and we get **Wednesday**.

4. What is the average of all possible 3-digit numbers formed by the digits 1, 9, and 8 without repeated digit?

【Answer】666

【Solution】

Each digit appears $2 \times 1 = 2$ times on hundreds place, tens place and ones place. Th sum of all numbers is $(1+9+8) \times 111 \times 2 = 3996$. Since there is a total of $3 \times 2 \times 1 = 6$ numbers, the average is $3996 \div 6 = 666$.

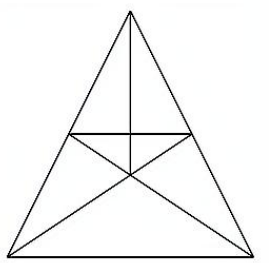
5. A car travels from point A to point B at 100 km/h, then returns along the same route at 60 km/h. Find the car's average speed for the whole trip.

【Answer】75 km/h

【Solution】

We know the speed to complete the same route but the distance is unknown. Hence, we try to set a specific distance for the route. To make calculations easy, choose a distance that is a common multiple of 100 and 60, such as 300 km.
Time from A to B is $300 \div 100 = 3$ hours.
Time from B to A is $300 \div 60 = 5$ hours.
Total time is 3 + 5 = 8 hours.
Total distance is 300 + 300 = 600 km.
Average speed is $600 \div 8 = 75$ km/h

6. Find the number of triangles of the figure.

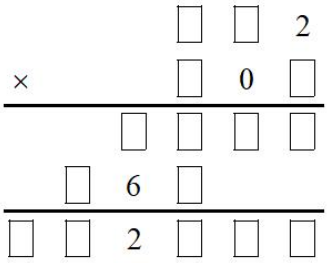


【Answer】20

【Solution】

Counting from the smallest to the largest
Triangles made of 1 part: 7
Triangles made of 2 parts: 6
Triangles made of 3 parts: 4
Triangles made of 5 parts: 2
Triangles made of 7 parts: 1
7 + 6 + 4 + 2 + 1 = 20 triangles

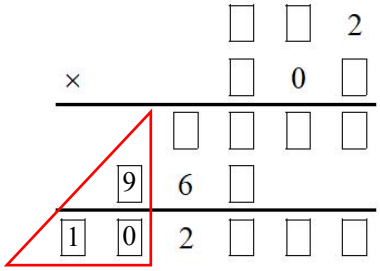
7. Fill in the blanks in the multiplication formulae to complete. What is the sum of the two 3-digit number that are multiplied together?



【Answer】1069

【Solution】

Using the Golden Triangle, we can find out the 3 digits in front are 1, 9 and 0.



From the 3rd row, the number can be $\overline{5\square\square}$ or $\overline{6\square\square}$.

From the 1st, 2nd and 4th row, we know that $a\times\overline{b\square 2}=\overline{96\square}$.

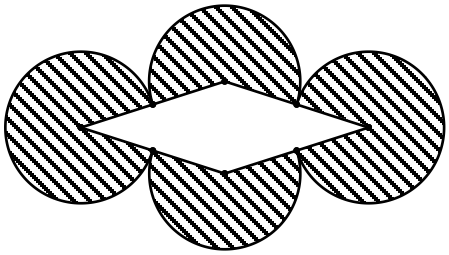
If $a\geq 2$, then $b\leq 4$. This way we cannot make the first digit on the 3rd row to be 5 or 6.

Therefore, $a=1$. The numbers on the 1st and 4th row are both 962.

Upon checking, the number on the 2nd row is 107. Sum of the two multiplicands is $962+107=1069$

Questions 8 to 14 are worth 6 marks each.

8. As shown in the figure, the side length of the rhombus is 20, draw 4 circle with a radius of 10 and the 4 vertices as the center, find the sum of the area of the shaded part. (Take $\pi = 3.14$)



【Answer】942

【Solution】

Sum of interior angles of the 4 sectors is $4\times 360°-(4-2)\times 180°=3\times 360°=1080°$.

Area of the shaded parts is $\frac{1080°}{360°}\times\pi\times 10^2=300\pi=942$.

9. For an any number *x,* $\lceil x \rceil$ denoted as the smallest whole number no less than *x*, for example $\lceil 7.23 \rceil = 8$, $\lceil 9 \rceil = 9$. Find the value of $\left\lceil \frac{1}{41} \right\rceil + \left\lceil \frac{2}{41} \right\rceil + \ldots + \left\lceil \frac{2025}{41} \right\rceil$.

【Answer】51025

【Solution】

Let each term be $\left\lceil \frac{n}{41} \right\rceil$. $1 \le n \le 2025$ and n is an integer.

When $1 \le n \le 41$, $\left\lceil \frac{n}{41} \right\rceil = 1$.

When $42 \le n \le 82$, $\left\lceil \frac{n}{41} \right\rceil = 2$.

… …

Thus, every 41 terms are in 1 group. The value of $\left\lceil \frac{n}{41} \right\rceil$ is 1, 2, 3, …

Since $2025 = 41 \times 49 + 16$, sum of all terms is

$= 41 \times (1 + 2 + \cdots + 49) + 16 \times 50$

$= 41 \times 50 \times 49 \div 2 + 32 \times 25$

$= 2041 \times 25$

$= 51025$

10. Use the 2026 numbers 2, 3, 4,..., 2026,2027 as numerators and denominators to construct 1013 fractions. The minimum value of the largest fraction among these 1013 fractions is $\frac{b}{a}$. Find the value of $a + b$.

【Answer】3041

【Solution】

(1) Let the larger half of the numbers as the denominator and the smaller half of the numbers as the numerator. Arrange in an increasing order: $\frac{2}{1015}$, $\frac{3}{1016}$, …, $\frac{1014}{2027}$. Based on concentration principle, the largest fraction is $\frac{1014}{2027}$.

(2) If we want to find a largest fraction that is smaller than $\frac{1014}{2027}$, then 2027 cannot be the numerator and it needs to pair with a numerator smaller than 1014. Considering 1014, 1015, …, 2026, these 1013 numbers will have 1 number that must be a numerator and pair with a denominator smaller than 2027. This result to a fraction that is larger than $\frac{1014}{2027}$.

Hence, the minimum value of the largest fraction is $\frac{1014}{2027}$,

the sum of $a + b = 1014 + 2027 = 3041$

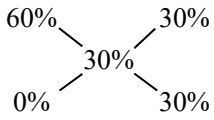
11. A student is conducting a controlled experiment to investigate the relationship between auxin concentration and plant growth. He plans to prepare 2026 grams of 10%,20%,30%,40%, and 50% auxin solutions using 60% auxin and distilled water. Find the total amount of distilled water required for preparation (in gram).

【Answer】5065

【Solution】

Total weight of 10%,20%,30%,40%, and 50% auxin solutions is (5×2026) grams. The overall concentration is $\dfrac{10\%+20\%+30\%+40\%+50\%}{5}=30\%$.

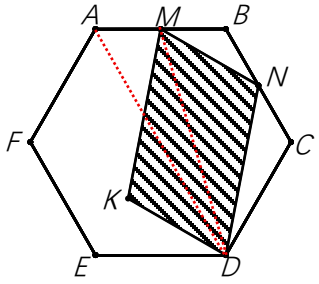
Using the Cross Method, we have

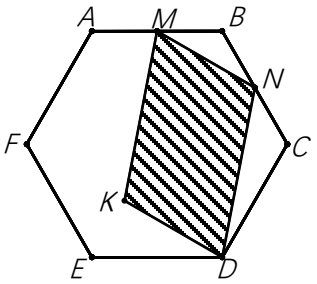


Thus, the ratio of 60% auxin solution and distilled water used is $30\%:30\%=1:1$.

Amount of distilled water used is $(5\times 2026)\times\dfrac{1}{1+1}=5065$ grams.

12. As shown in the figure, in the regular hexagon *ABCDEF*, *M* and *N* are the midpoints of sides *AB* and *BC*, respectively. If the area of the parallelogram *MNDK* is 125, what is the area of the regular hexagon *ABCDEF*?





【Answer】300

【Solution】

Connect *DA* and *DM*.

(1) $S_{\triangle MND}=\dfrac{1}{2}\times S_{\square MNDK}=\dfrac{125}{2}$

(2) Since *M* and *N* are the midpoints of *AB* and *BC*,

$$S_{\triangle NCD}=\frac{1}{2}\times\frac{1}{6}\times S_{ABCDEF}$$

$$S_{\triangle MND}=\frac{1}{2}\times S_{ABND}=\frac{1}{2}\times\left(\frac{1}{2}\times S_{ABCDEF}-S_{\triangle NCD}\right)=\frac{5}{24}S_{ABCDEF}$$

$$S_{ABCDEF}=S_{\triangle MND}\div\frac{5}{24}=\frac{125}{2}\times\frac{24}{5}=300$$

13. Ivan and Peter start from points A and B respectively, with Ivan walking at 60 meters per minute and Peter at 36 meters per minute. If they set off at the same time, they meet at point C. If Ivan had started 10 minutes later, they would have met at point D. Given that the distances from the midpoint of AB to C and D are equal, what is the distance between points A and B?

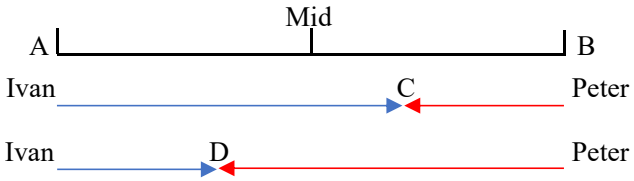
【Answer】900

【Solution】

Speed ratio of Ivan to Peter is $60:36=5:3$.

Thus, distance ratio of Ivan to Peter under the same time duration is also $5:3$.

When they set off at the same time, we can let the distance travelled by Ivan be 5u; distance travelled by Peter be 3u.



Since the distance from the midpoint to C is equal to the distance from the midpoint to D, we know that Ivan travelled $3u$ and Peter travelled $5u$ the second time. Under the same time duration, when Ivan has travelled $3u$, Peter can travel $3\div5\times3=1.8u$.

For the second time, Peter travelled $10\times36=360$ m before Ivan started. This distance equals to $5u-1.8u=3.2u$.

$1u=360\div3.2=112.5$

Total distance is $8u=8\times112.5=900$ m.

14. Emma and Frank are playing an arithmetic game. Emma first selects a whole number less than 2026 and writes it on the blackboard. Frank then takes turns with Emma, following this sequence: Frank erases the number and writes the sum of the 2025 times of it and 3; Emma erases the number and writes the sum of the 2027 times of it and 4. The game ends when either player's result becomes a multiple of 2026. How many different ways could Emma have chosen the number initially to keep the game going?

【Answer】2022

【Solution】

Let the number chosen by Emma be n. Then, as Emma and Frank take turns performing the operation, the remainders when the resulting numbers are divided by 2026 follow the pattern $n,\ 3-n,\ 7-n,\ n-4,\ n,\ \ldots$

That is, the remainders repeat in a cycle of four.

Therefore, for the game to continue, n cannot be 0, 3, 4 or 7. Hence, there are 2022 possible choices that allow the game to continue.

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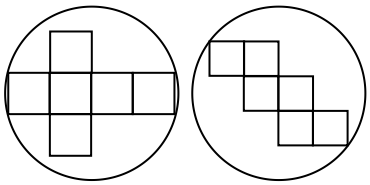
Questions 15 to 19 are worth 8 marks each.

15. As shown in the figure, two identical circular paper sheets are cut into a net unfolding of a maximum cube as illustrated. If the side length of the smaller square on the left is 10, what is the area of the smaller square on the right?



【Answer】68

【Solution】

As shown in the figure, AC and DE are the diameter, Let the side length of each small square in the right circle be a.

Using Pythagorean theorem

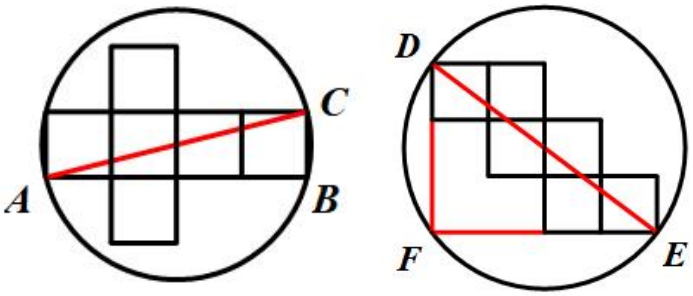
$AC^2 = AB^2 + BC^2 = 40^2 + 10^2 = 1700$

$DE^2 = DF^2 + EF^2 = (3a)^2 + (4a)^2 = 25a^2$

So $25a^2 = 1700$

$25a^2 = 1700$

$a^2 = 68$



16. Given that the positive integer M is a common multiple of 12,18 and 24, and in its prime factorization, each prime factor has an odd power. If the sum of all factors of M is 32760, what is the value of M?

【Answer】9720

【Solution】

$HCF(12,18,24) = 2^3 \times 3^2$, $32760 = 2^3 \times 3^2 \times 5 \times 7 \times 13$

Let $M = 2^a \times 3^b \times x^c \times y^d \times \cdots$, where x, y,... are prime numbers other than 2 and 3, and a, b, c,... are odd numbers, $a \geq 3$, $b \geq 3$.

The sum of all factors of M is $(1+2+\cdots+2^a) \times (1+3+\cdots+3^b) \times (1+x+\cdots+x^c) \times \cdots$

If $a = 3$, $1+2+2^2+2^3 = 15 = 3 \times 5$,

If $a = 5$, $1+2+\cdots+2^5 = 63 = 3^2 \times 7$

If $b = 3$, $1+3+3^2+3^3 = 40 = 2^3 \times 5$

If $b = 5$, $1+3+\cdots+3^5 = 364 = 2^2 \times 7 \times 13$

Since 32760 has only one prime factor, 5, a and b cannot both be 3.

If $a = 3$, $b = 5$, $32760 \div 15 \div 364 = 6$

So $x = 5$, $c = 1$, $M = 2^3 \times 3^5 \times 5^1 = 9720$

If $a = 5$, $b = 3$, $32760 \div 63 \div 40 = 13$

No prime number satisfies the condition.

If $a \geq 7$ or $b \geq 7$, no solution.

Therefore, $M = 9720$

17. A box contains marbles numbered from 1 to 7. Eight marbles are drawn with replacement, and only the numbers are recorded. How many different combinations of numbers are possible if the order does not matter? (12345677 and 77654321 is the same)

【Answer】3003

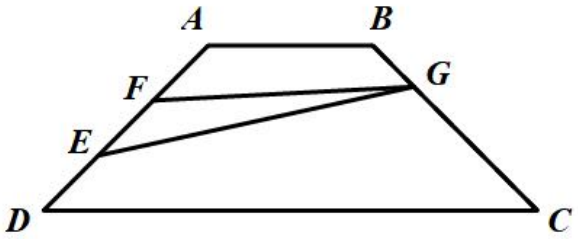
【Solution】

We are selecting 8 items from 7 distinct types (the numbers 1 through 7), and the order of selection does not matter.

$8+7=15$, we can consider this question as "selecting 15 digit from 1 to 7 with replacement, with each digit at least 1" , using insetting bar method.

$$14C6=\frac{14\times13\times12\times11\times10\times9}{6\times5\times4\times3\times2\times1}=3003$$

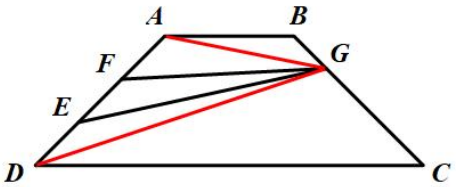
18. As shown in the figure, in the isosceles trapezium *ABCD*, the top base *AB* is 4 cm, the bottom base *CD* is 12 cm, and the angle between the leg *AD* and the base *DC* is 45°. If *AF* = *FE* = *ED* and *BC* = 4*BG*, what is the area of triangle *EFG*?



【Answer】4

【Solution】

Connect AG and DG, using the Equal-Height Model, the area of EFG $=\frac{1}{3}$ the area of ADG.

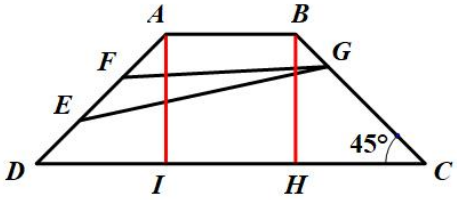


the area of $ADG=ABCD-ABG-CDG$

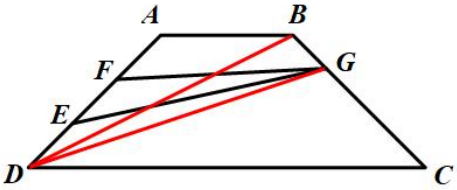
Since the base angle is 45°, the triangle *BCH* and *ADI* are isos. right triangle.

So $AI=DI=BH=CH=(12-4)\div2=4$

The area of $ABCD=(4+12)\times4\div2=32$



Connect BD, using the Equal-Height Model, the area of $CDG=\frac{3}{4}$ of the area of $BCD=\frac{3}{4}\times12\times4\div2=18$



Similarly, the area of $ABG=\frac{1}{4}$ of the area of $ABC=\frac{1}{4}\times4\times4\div2=2$

Finally, the area of $ADG=32-18-2$

the area of $EFG=\frac{1}{3}ADG=\frac{1}{3}\times12=4$

19. The king has forgotten the password to his treasure vault. The password consists of 8 letters from 'Kangaroo' and any 2 adjacent digits are distinct. What is the number of possible passwords?

【Answer】5760

【Solution】

Apply the Principle of Inclusion-Exclusion

1. Total permutations without any restrictions:
Calculate the number of distinct permutations of the 8 letters, accounting for the duplicates.

$$\frac{8!}{2!\cdot 2!}=\frac{40320}{4}=10080$$

2. Subtract the invalid arrangements where at least one pair of identical letters is adjacent:
- Case 1: The two A's are adjacent.
Treat "AA" as a single entity. We now have 7 "items" to arrange: {AA, K, N, G, R, O, O}.

$$\frac{7!}{2!}=\frac{5040}{2}=2520$$

- Case 2: The two O's are adjacent.
Treat "OO" as a single entity. We now have 7 "items" to arrange: {OO, K, N, G, R, A, A}.

$$\frac{7!}{2!}=\frac{5040}{2}=2520$$

3. Add back the arrangements where both pairs are adjacent (they were subtracted twice):
Treat both "AA" and "OO" as single entities. We now have 6 "items" to arrange: {AA, OO, K, N, G, R}.

$$6!\ =\ 720$$

4. Calculate the number of valid arrangements:

$$10080-2520-2520+720=5760$$

Question 20 is worth 10 marks.

20. In your opinion, from question 1 to 19, your favourite question is question _____ and the most difficult question is question _____.
(As long as your answer is within 1 to 19, you get full marks, otherwise you get zero.)