2023 Spring Cup
Mathematical Olympiad
PRELIMINARY ROUND

Date: 28 January 2023

Time Given: 1 hour 30 minutes

Level: Primary 6

Name: _____

Instructions to Candidates

1. Do not open the booklet until you are told to do so.
2. Answer ALL 20 questions.
3. Write your answers in the answer sheet provided.
4. No steps are needed to justify your answers.
5. Questions 1-7 are worth 4 marks each.
6. Questions 8-14 are worth 6 marks each.
7. Questions 15-19 are worth 8 marks each.
8. Question 20 is worth 10 marks.
9. No marks will be deducted for wrong answers.
10. No marks will be given for unanswered questions.
11. No calculators or mathematical instruments are allowed.

Questions 1 to 7 are worth 4 marks each.

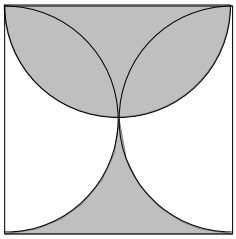
1. A new operation symbol ※ is defined as $a※b=3a-2b$. If $x※(4※1)=7$, then x=_____.

2. The numbers 1 to 65 are arranged according to the figure below. After removing the multiples of 6, there are 55 numbers left. What is the sum of the remaining 55 numbers?

1	7	13	19	25	31	37	43	49	55	61
2	8	14	20	26	32	38	44	50	56	62
3	9	15	21	27	33	39	45	51	57	63
4	10	16	22	28	34	40	46	52	58	64
5	11	17	23	29	35	41	47	53	59	65

3. In a charity, Adam, Bob and Carla donated 80 dollars in total. If Adam donated 18 dollars more than Carla and the ratio between the sum of donation of Adam and Bob, and the sum of donation of Bob and Carla is 10 : 7, how much did Carla donate?

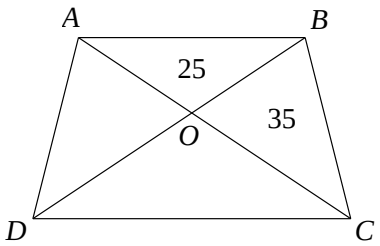
4. As shown in the figure, three semicircles were drawn in a square with a side of 2cm. If the diameter of the semicircles is the side of the square, the area of the shaded region is _____ cm^2.



5. How many ways are there to pick two numbers from numbers 1 to 40 such that the sum of the two numbers is divisible by 4?

6. Alice bought 10 kg of mushrooms. The mushrooms contain 99% water. After putting it under the sun, the mushrooms now contain 98% water. The weight of water that has evaporated is _____ kg.

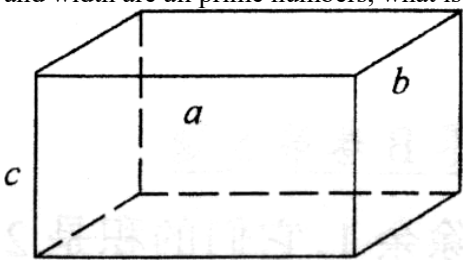
7. As shown in the figure, $ABCD$ is a trapezium with two parallel lines AB and CD, the diagonal AC and BD intersects at point O. If the area of $\triangle AOB$ and $\triangle BOC$ is 25 cm^2 and 35 cm^2 respectively, the area of trapezium $ABCD$ is _____ cm^2



Questions 8 to 14 are worth 6 marks each.

8. Jockey is writing some digits after the number 1989, he follows the rule that the next number he writes down is the ones place of the product of the previous two numbers (for example, $8\times 9=72$, so Jockey writes down 2 after 9, $9\times 2=18$, so Jockey writes down 8 after 2). Since the first 4 digits are 1, 9, 8 and 9, from left to right, what is the sum of the first 1999 digits?

9. As shown in the figure, the sum of the area of a cuboid's front and top is 209. If its length, height and width are all prime numbers, what is the volume of this cuboid?



10. $\dfrac{\frac{1}{2}}{1+\frac{1}{2}}+\dfrac{\frac{1}{3}}{(1+\frac{1}{2})\times(1+\frac{1}{3})}+\cdots+\dfrac{\frac{1}{1999}}{(1+\frac{1}{2})\times(1+\frac{1}{3})\times\cdots\times(1+\frac{1}{1999})}=$_____

11. Three remainders can be obtained after dividing a whole number N by 70, 110 and 160, if the sum of the three remainders is 50, what is the number N?

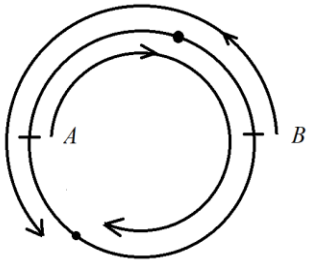
12. Student A, B, C, D and E entered a drawing competition. After the competition, student A and B went to ask the judge about the results, the judge told student A: “Unfortunately, you and student B are not the champion” and he told student B “You’re not in the last place”. Based on this information, how many possible ways are there to rank the five students?

13. There are 25 tokens labelled with an odd number from 1 to 49. Two pupils each take turns to take a token. If one of them take the token with label x, the next person must take the token labelled with the greatest odd number factor of $99 - x$. If the first pupil takes the token with label 5, how many tokens would be left when the game ends?

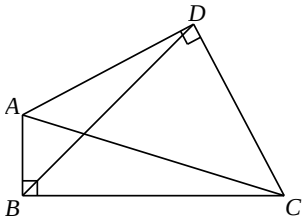
14. 100 cards were placed on the floor written with the number 1 to 100 and none of the number repeats. At least how many cards must be drawn to ensure the product of the number on the drawn cards is divisible by 12?

Questions 15 to 19 are worth 8 marks each.

15. Aaron and Ben started running from two opposite points on the diameter of a circular running track. Aaron runs in the clockwise direction and Ben runs in the anti-clockwise direction. When Ben has ran for 100 meters, they meet for the first time. They meet for the second time when Aaron left 60 meters to reaching his starting point. What is the perimeter of this circular track?

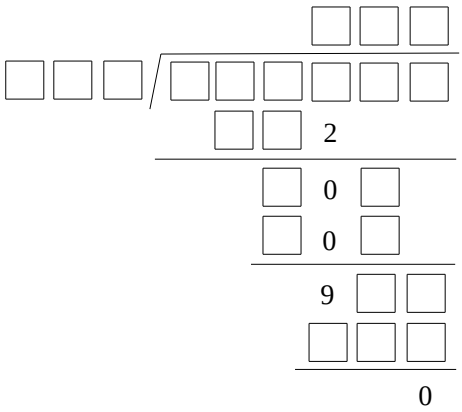


16. As shown in the figure below, $\angle ABC$ and $\angle ADC$ are two right angles in the quadrilateral $ABCD$. If $AD = DC$, $BD = 6$ and $AC = 8$, find the area of the triangle ABC.



17. Port A and B are 15 km apart along a river. Port A is upstream and port B is downstream. A ferry and a boat start travelling downstream from port A to port B respectively at the same time. After 5 hours, the ferry catches up with the boat. After travelling for another 1 hour, the ferry dropped an item into the river (the item floats on the river and does not move). The crew on the ferry only realized this after 6 minutes and they quickly made a turn to look for the item. When they reach the item, they meet again with the boat. What is the speed of the boat in km/h?

18. Complete the following vertical algorithm by filling in the blanks. If we know the quotient is an odd number, what is the dividend?



19. At least how many cubes with edge of length 1 does it take to form a cuboid with surface area 52?

Question 20 is worth 10 marks.

In your opinion, from question 1 to 19, your favourite question is question _____ and the most difficult question is question .

(As long as your answer is within 1 to 19, you get full marks, otherwise you get zero.)