



**2024 Spring Cup**  
**Mathematical Olympiad**  
**PRELIMINARY ROUND**

Date: 28 January 2024

Time Given: 1 hour

Level: Primary 4&5

Name: \_\_\_\_\_

**Instructions to Candidates**

1. Do not open the booklet until you are told to do so.
2. Answer ALL 20 questions.
3. Write your answers in the answer sheet provided.
4. No steps are needed to justify your answers.
5. Questions 1-7 are worth 4 marks each.
6. Questions 8-14 are worth 6 marks each.
7. Questions 15-19 are worth 8 marks each.
8. Question 20 is worth 10 marks.
9. No marks will be deducted for wrong answers.
10. No marks will be given for unanswered questions.
11. No calculators or mathematical instruments are allowed.

Questions 1 to 7 are worth 4 marks each.

1.  $2024 \times \left( \frac{1}{11} + \frac{2}{23} \right)$

【解析】原式 =  $2024 \times \frac{1}{11} + 2024 \times \frac{2}{23} = 184 + 176 = 360$

2. The sum of three whole numbers  $A$ ,  $B$  and  $C$  is 138. Suppose that  $A$  is 20% more than  $B$  while  $B$  is 20% less than  $C$ . Find the value of  $C$ .

【解析】 $A:B=6:5$ ,  $B:C=4:5$ ,  $A:B:C=24:20:25$ ,  $C=138 \div 69 \times 25=50$

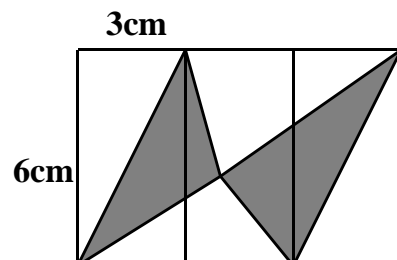
3. It is known that the whole number in each box below is the same.

$$\square \times \square + \square = 870$$

What is the whole number?

【解析】 $x(x+1)=870$ ,  $870=2 \times 3 \times 5 \times 29=30 \times 29$ . Ans: 29

4. In the figure below, three  $3 \text{ cm} \times 6 \text{ cm}$  rectangles are placed together in a row. Find the area (in  $\text{cm}^2$ ) of the unshaded region.



【解析】 $3 \times 6 + 6 \times 6 \div 2 = 36$

5. The sum of the father and son's ages is 45 years. 5 years from now, father's age will be exactly 4 times that of the son. What is the current age of father?

【解析】再过5年，父子俩一共长了10岁，那时他们的年龄之和是 $45 + 10 = 55$  (岁)，由于父亲的年龄是儿子的4倍，因而55岁相当于儿子年龄的 $4 + 1 = 5$ 倍，可以先求出儿子5年后的年龄，再求出他们父子今年的年龄。

5年后的年龄和为： $45 + 5 \times 2 = 55$  (岁)

5年后儿子的年龄： $55 \div (4 + 1) = 11$  (岁)

儿子今年的年龄： $11 - 5 = 6$  (岁)，父亲今年的年龄： $45 - 6 = 39$  (岁)

6. There are 100 numbers in a row, as shown:

$$a_1, a_2, a_3, \dots, a_{100}$$

Given that the average of these 100 numbers is 95, while the average of the first 51 numbers is 100 and the last 50 numbers is 90. Find the value of  $a_{51}$ .

【解析】 $51 \times 100 + 50 \times 90 - 100 \times 95 = 100$

7. A train with a length of 200 meters passes through a tunnel. The time between the train head to enter and the train tail to exit the tunnel is 60 seconds. The time for the train to be completely inside the tunnel is 40 seconds. What is the speed of the train?

【解析】从火车进隧道到完全出来用60秒走的路程 = 桥长 + 火车长，完全在隧道中的时间40秒走的路程 = 桥长 - 火车长，可知60秒比40秒多20秒，走的路程多两个火车长，即一个车长用时间为 $20 \div 2 = 10$  (秒)。车长为200米，所以车速： $200 \div 10 = 20$  (米/秒)。

Questions 8 to 14 are worth 6 marks each.

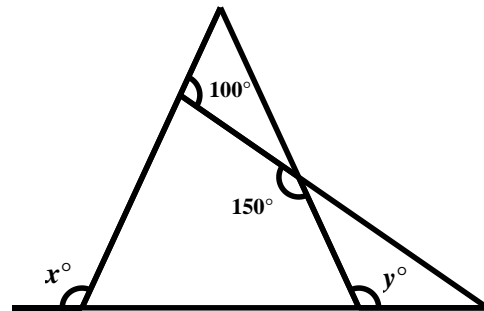
8. Benjamin drove a car from city  $A$  to city  $B$  with constant speed. If he drove 10% faster than this speed, he will reach city  $B$  twenty minutes earlier. How long, in minutes, did Benjamin spend for his journey, using his initial speed?

【解析】速度的比是 10:11,所以时间比是 11:10,  $20 \times 11 = 220$ min

9. A glass with water weighs 172 grams when it is one-third full and 192 grams when it is half full. What is the weight, in grams, of a full glass of water (including the glass)?

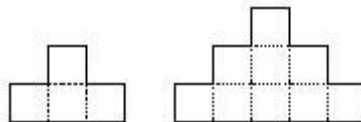
【解析】 $(192 - 172) \div \frac{1}{6} = 120, 120 + 132 = 252$ .

10. In the figure below, find the value of  $x + y$ .



【解析】 $x + y = 180^\circ + 50^\circ = 230^\circ$

11. The two figures in the diagram (solid lines) are each surrounded by 10 and 16 sticks, respectively. If following this pattern (each layer has two additional small squares compared to the layer above) a figure surrounded by 64 sticks is formed, how many layers does the figure have?



【解析】通过观察每增加一层,恰好增加 6 根小棍,这 6 根恰好是增加那一层比上一层多摆出的两个正方形多用的,即前 1 层用 4 根,前 2 层用  $4 + 6$  根,前 3 层用  $4 + 6 \times 2$  根,前  $n$  层用  $4 + 6 \times (n - 1)$  根,所以共用小棍 64 根,围成的图形有 11 层。

12. How many four-digit numbers, where the thousands digit is 8 and each number has exactly two identical digits (e.g., 8336, 8545, 8782), are there?

【解析】两个相同的数字是 8 时，另一个 8 有 3 个位置可选，其余两个位置有  $9 \times 8 = 72$  种填法，有  $3 \times 9 \times 8 = 216$  个数；

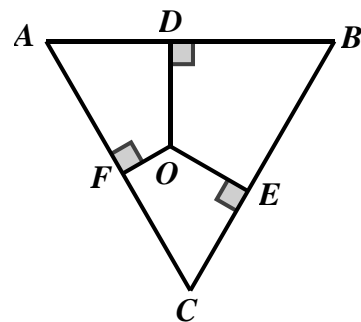
两个相同的数字不是 8 时，相同的数字有 9 种选法，不同的数字有 8 种选法，并有 3 个位置可放，有  $9 \times 8 \times 3 = 216$  个数。

由加法原理，共有  $3 \times 9 \times 8 + 9 \times 8 \times 3 = 432$  个数。

13. In a bag, there are black and white balls. After removing 15 white balls, black balls constitute  $\frac{2}{3}$  of the total. After removing 45 black balls, black balls constitute  $\frac{1}{6}$  of the total. Find the initial number of black and white balls.

【解析】第二次拿走 45 黑，黑与白的个数之比由  $2:1 (=10:5)$  变为  $1:5$ ，而其中白的数目是不变的，所以黑由原来的 10 份变成现在的 1 份，减少了 9 份，这样原来黑的个数为  $45 \div 9 \times 10 = 50$  (枚)，白的个数为  $45 \div 9 \times 5 + 15 = 40$  (枚)。一共 90 枚。

14. The diagram shows an equilateral triangle ABC with OD, OE and OF perpendicular to AB, BC and CA respectively. If  $OD+OE+OF=28\text{cm}$  and the height of triangle ABC =  $x$  cm, find the value of  $x$ .



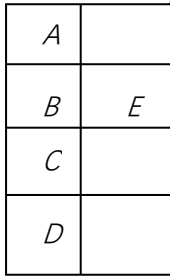
【解析】28

Questions 15 to 19 are worth 8 marks each.

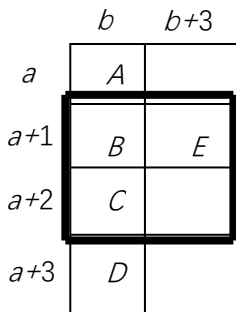
15. What is the average of all 4-digit numbers that can be formed by using each of the digits 2, 4, 6 and 8 exactly once?

【解析】 $(2468+8642) \div 2=5555$

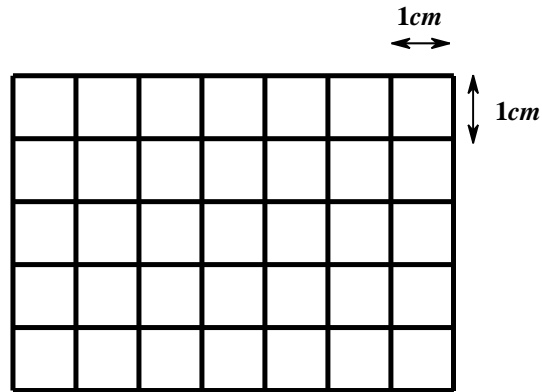
16. Given the figure, a rectangle is divided into 8 smaller rectangles, with the perimeters of rectangles A, B, C, D, and E being 26 cm, 28 cm, 30 cm, 32 cm, and 34 cm respectively. What is the maximum possible area of the largest rectangle in square centimeters?



【解析】设长方形 A 的长和宽分别为  $b$  和  $a$ , 则长方形 B、C、D、E 的长和宽如图所示. 由于  $2a + 2b = 26$ , 所以中间两行四个长方形组成的中间长方形的长宽之和等于  $(2a + 3) + (2b + 3) = 32$ , 长宽的乘积最大为  $16 \times 16 = 256$ . 从图上可以看出大长方形的宽等于中间长方形的 2 倍, 长与中间长方形相等, 因此大长方形面积等于中间长方形面积的 2 倍, 于是大长方形的面积最大是 512 平方厘米.



17. The figure below shows a  $7 \times 5$  grid. Suppose that the side length of each small square is 1 cm. What is the sum of the areas of all rectangles with a perimeter of 12? (The square is also a rectangle.)



【解析】 $1 \times 5: 3 \times 5 = 15$ ;  $2 \times 4: 4 \times 4 = 16$ ;  $3 \times 3: 5 \times 3 = 15$ ;  $4 \times 2: 6 \times 2 = 12$ ;  $5 \times 1: 7 \times 1 = 7$   
 $15 \times 5 + 16 \times 8 + 15 \times 9 + 12 \times 8 + 7 \times 5 = 469$

18. Abel, Ben and Calvin completed a project together in 13 days. For the first 6 days, Abel and Ben completed  $\frac{1}{3}$  of the project together. For the next 2 days, Ben and Calvin completed  $\frac{1}{6}$  of the project together. For the last 5 days, Abel, Ben and Calvin completed the rest of the project together. Ben was paid \$1820, which was distributed among them according to the amount of work they did in this project. How much money should be given to Abel?

【解析】根据题意可知，Abel、Ben 两人的工作效率之和为  $\frac{1}{3} \div 6 = \frac{1}{18}$ ；

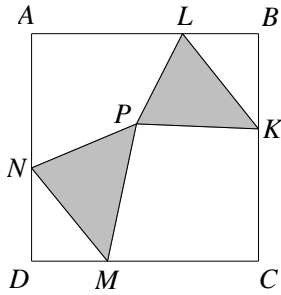
Ben、Calvin 两人的工作效率之和为  $\frac{1}{6} \div 2 = \frac{1}{12}$ ；

Abel、Ben、Calvin 三人的工作效率之和为  $\frac{1}{2} \div 5 = \frac{1}{10}$ 。

分别可求得 Abel 的工作效率为  $\frac{1}{10} - \frac{1}{12} = \frac{1}{60}$ ，Ben 的工作效率为  $\frac{1}{18} - \frac{1}{60} = \frac{7}{180}$ ，Calvin 的工作效率为  $\frac{1}{10} - \frac{1}{18} = \frac{2}{45}$ ，则 Abel 完成的工程量为： $\frac{1}{60} \times (6+5) = \frac{11}{60}$ ，Ben 完成的工程量为： $\frac{7}{180} \times (6+2+5) = \frac{91}{180}$ ，Calvin 完成的工程量为： $\frac{2}{45} \times (2+5) = \frac{14}{45}$ ，三人所完成的工作量之比为  $\frac{11}{60} : \frac{91}{180} : \frac{14}{45} = 33:91:56$ 。

所以，Abel 应得  $1820 \div 91 \times 33 = 660$  元。

19.  $ABCD$  is a square with a side length of 12, as shown in the diagram.  $P$  is a random point inside, and  $BL = DM = 4$ 、 $BK = DN = 5$ . Find the area of the shaded region.



【解析】(法1) 特殊点法. 由于  $P$  是内部任意一点, 不妨设  $P$  点与  $A$  点重合(如上中图), 那么阴影部分就是  $\triangle AMN$  和  $\triangle ALK$ . 而  $\triangle AMN$  的面积为  $(12-5) \times 4 \div 2 = 14$ ,  $\triangle ALK$  的面积为  $(12-4) \times 5 \div 2 = 20$ , 所以阴影部分的面积为  $14 + 20 = 34$ .

(法2) 寻找可以利用的条件, 连接  $AP$ 、 $BP$ 、 $CP$ 、 $DP$  可得右上图所示:

$$\text{则有: } S_{\triangle PDC} + S_{\triangle PAB} = \frac{1}{2} S_{ABCD} = \frac{1}{2} \times 12^2 = 72$$

$$\text{同理可得: } S_{\triangle PAD} + S_{\triangle PBC} = 72;$$

$$\text{而 } S_{\triangle PDM} : S_{\triangle PDC} = DM : DC = 4 : 12 = 1 : 3, \text{ 即 } S_{\triangle PDM} = \frac{1}{3} S_{\triangle PDC};$$

$$\text{同理: } S_{\triangle PBL} = \frac{1}{3} S_{\triangle PAB}, S_{\triangle PND} = \frac{5}{12} S_{\triangle PDA}, S_{\triangle PBK} = \frac{5}{12} S_{\triangle PBC};$$

$$\text{所以: } (S_{\triangle PDM} + S_{\triangle PBL}) + (S_{\triangle PND} + S_{\triangle PBK}) = \frac{1}{3} (S_{\triangle PDC} + S_{\triangle PAB}) + \frac{5}{12} (S_{\triangle PDA} + S_{\triangle PBC})$$

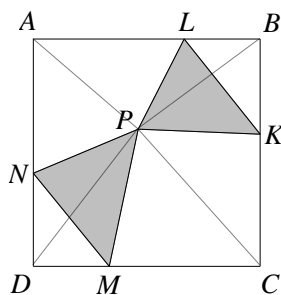
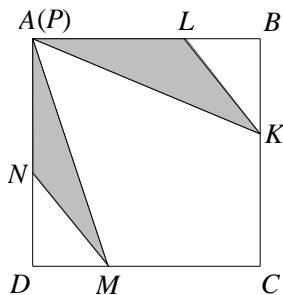
$$\text{而 } (S_{\triangle PDM} + S_{\triangle PBL}) + (S_{\triangle PND} + S_{\triangle PBK}) = \underbrace{(S_{\triangle PNM} + S_{\triangle PLK})}_{\text{阴影面积}} + (S_{\triangle DNM} + S_{\triangle BLK});$$

$$S_{\triangle DNM} = S_{\triangle BLK} = \frac{1}{2} \times 4 \times 5 = 10;$$

所以阴影部分的面积是:

$$S_{\triangle PNM} + S_{\triangle PLK} = \frac{1}{3} (S_{\triangle PDC} + S_{\triangle PAB}) + \frac{5}{12} (S_{\triangle PDA} + S_{\triangle PBC}) - (S_{\triangle DNM} + S_{\triangle BLK})$$

$$\text{即为: } \frac{1}{3} \times 72 + \frac{5}{12} \times 72 - 10 \times 2 = 24 + 30 - 20 = 34.$$



Question 20 is worth 10 marks.

20. In your opinion, from question 1 to 19, your favourite question is question \_\_\_\_\_ and the most difficult question is question \_\_\_\_\_.

(As long as your answer is within 1 to 19, you get full marks, otherwise you get zero.)