数学花园探秘
MATHEMATICS GARDEN

2026 Spring Cup
Mathematical Olympiad

Date: 28 February 2026

Time Given: 1 hour 30 minutes

Level: Secondary Advanced

Name: _____

Parent's Phone Number: _____

Instructions to Candidates

1. Do not open the booklet until you are told to do so.
2. Answer ALL 13 questions.
3. Write your answers in the answer sheet provided.
4. Questions 9(a), 12, 13 require full working steps.
5. Questions 1-4 are worth 8 marks each.
6. Questions 5-8 are worth 10 marks each.
7. Questions 9-11 are worth 14 marks each.
8. Questions 12-13 are worth 18 marks each.
9. No marks will be deducted for wrong answers.
10. No marks will be given for unanswered questions.
11. No calculators or mathematical instruments are allowed.

I. ***Short Answer Questions(1) (8 marks each, 32 marks in Total)***

1. Given that for any $x \in \mathrm{R}$, the function $f(x)$ satisfies $f(x+1)=\dfrac{f(x)-1}{f(x)+1}$, and $f(1) = -2$, find the value of $f(2026)$.

Ans: _____

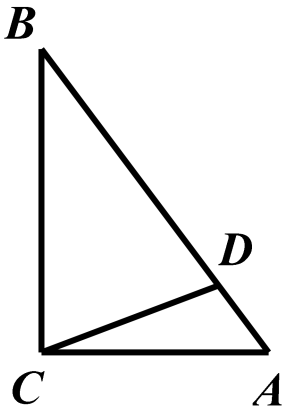
2. Let x and y be positive real numbers. What is the smallest possible value of $\dfrac{16}{x}+\dfrac{108}{y}+xy$?

Ans: _____

3. The result of the expression $\sqrt[3]{26+15\sqrt{3}}-\sqrt[3]{-26+15\sqrt{3}}$ is _____.

Ans: _____

4. As shown in the figure, in a right triangle ABC, $\angle C = 90°$, $AC = 6$, $BC = 8$, point D is on the hypotenuse AB, and $\angle CDB = 2\angle B$, find the length of CD.



Ans: _____

II. ***Short Answer Questions(2) (10 marks each, 40 marks in Total)***

5. Find the units digit of $\left(2^1+1\right)\left(2^2+1\right)\left(2^3+1\right)...\left(2^{2017}+1\right)$.

Ans: _____

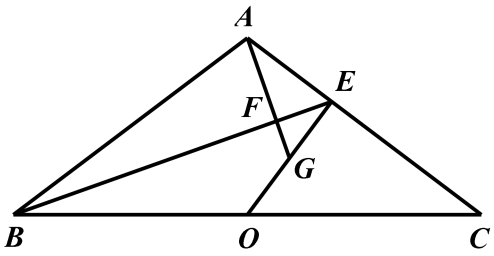
6. Given that two real numbers x, y satisfy $20x^2 - 14x - 5 = 0$, $y^2 + 26y + 20 = 0$, and $xy \neq y + 1$, then find the value of $\dfrac{xy - y + x}{y}$.

Ans: _____

7. Find the value of $\sin 6° \sin 42° \sin 66° \sin 78°$.

Ans: _____

8. As shown in the figure, in an isosceles triangle ABC, $AB = AC$, $BC = 20$, point O is the midpoint of BC, $OE \perp AC$ at point E, $AF \perp BE$ at point F and intersects OE at point G, $OG = 3$. Find the area of triangle $\triangle ABC$.



Ans: _____

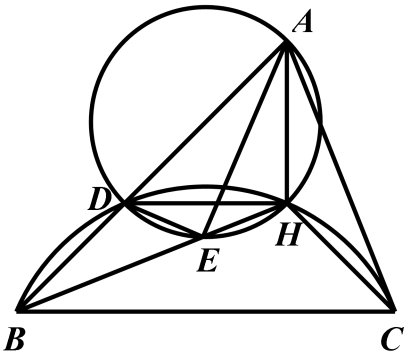
III. ***Short Answer Questions(3) (14 marks each, 42 marks in Total)***

9. On the six faces of a cube, each face is labeled with a distinct positive integer.
   The product of the numbers on two adjacent faces is written on their common edge, and the product of the numbers on the three faces meeting at a vertex is written at that vertex.
   Among these 26 numbers written,
   (a) At most how many different values can appear among these 26 numbers? **(7 marks)**

Ans: _____

   (b) At least how many different values must appear? Please explain your reasoning. **(7 marks)**

10. In an acute triangle ABC, $\angle ABC = 45°$ and H is the orthocenter. A circle passing through points B, H, and C intersects AB at point D. Another circle passing through points A, D, and H intersects BH at point E. Given that $DE = 2$ and $EH = 3\sqrt{2}$, find the length of EA.



Ans: _____

11. The inequality $a^2 + 4b^2 + 2n - 2025 \leq 2ab + 3a + 6b$ (where n is a positive integer) has exactly 16 integer solutions. What is the value of n?

Ans: _____

IV. ***Long Answer Questions (18 marks each, 36 marks in Total)***

12. Find all non-negative integer solutions (x, y, z) to the equation $2^x - 5^y \cdot 7^z = 1$.

13. Prove that if $\boldsymbol{p}$ is a prime number, then $p^2 \mid \left(\binom{2p}{p} - 2 \right)$.